Solow growth model

Labor productivity, $\gamma > 0, X_{t+1} = (1+\gamma)\,X_t \Rightarrow \frac{X_{t+1}}{X_t} = 1+\gamma, t = 0, 1, \ldots\ldots$

Population grow at rate of $n > 0$, $L_{t+1} = (1+n)L_t \Rightarrow \frac{L_{t+1}}{L_t} = 1+n$

Effective labor force $N_t = X_t\,L_t$

Production function $Y_t = A K_t^{\alpha} N_t^{1-\alpha}, 0 < \alpha < 1$

Consumption demand is a fraction of their income: $C_t = (1-s)\,Y_t$

Aggregate resource constraint: $C_t + I_t = Y_t$

Capital accumulation: $\delta = 1 \Rightarrow K_{t+1} = I_t$

$C_t + I_t = Y_t \Rightarrow I_t = Y_t - C_t \Rightarrow I_t = Y_t - (1-s)\,Y_t = s\,Y_t = K_{t+1}$

$\frac{N_{t+1}}{N_t} = \frac{X_{t+t}\,L_{t+1}}{X_t\,L_t} = (1+\gamma)\,(1+n)$

Efficiency unit of capital $k_t = \frac{K_t}{N_t}$, $k_{t+1} = \frac{K_{t+1}}{N_{t+1}}$

$K_{t+1} = s\,Y_t \Rightarrow \frac{K_{t+1}}{N_t} = s\frac{Y_t}{N_t}$

$\frac{K_{t+1}}{N_t} = \frac{K_{t+1}}{N_{t+1}}\frac{N_{t+1}}{N_t} = s\frac{Y_t}{N_t} \Rightarrow k_{t+1}(1+\gamma)\,(1+n) = s\frac{A K_t^{\alpha} N_t^{1-\alpha}}{N_t} = s\,A K_t^{\alpha} N_t^{-\alpha} = s\,A\left(\frac{K_t}{N_t}\right)^{\alpha} = s\,A k_t^{\alpha}$

$k_{t+1}(1+\gamma)\,(1+n) = s\,A k_t^{\alpha}$

In the steady state, $k_{t+1} = k_t = k$

$k(1+\gamma)\,(1+n) = s\,A k^{\alpha} \Rightarrow k^{1-\alpha} = \frac{s\,A}{(1+\gamma)\,(1+n)} \Rightarrow k = \left(\frac{s\,A}{(1+\gamma)\,(1+n)}\right)^{\frac{1}{1-\alpha}}$

Two economy: a and b,

Economy b has higher saving rate $s_b > s_a$ and higher labor productivity growth $\gamma_b > \gamma_a$

$\frac{s_b}{1+\gamma_b} = \frac{s_a}{1+\gamma_a}$

$k_a \geqslant < k_b$?

$k = \left(\frac{s\,A}{(1+\gamma)\,(1+n)}\right)^{\frac{1}{1-\alpha}}$