utility function: $u(C) + u(C')$

Time endowment: 1

Human capital accumulation requires "time": $1 - \phi$ of time goes to education, only ϕ of time goes to work.

Household's human capital endowment: H

human capital accumulation process: $H' = H + (1 - \phi) H$

Household's owns the capital and doing investment to accumulate capital.

physical capital endowment: K

physical capital accumulation process: $K' = (1 - \delta) K + I$

Production function: $Y = K^{\alpha}(\phi H)^{1-\alpha}; Y' = K'^{\alpha}(\phi' H')^{1-\alpha}$

Consumer owns the firm, i.e., claim the whole π

No government

Consumer's current budget constraint: $C \leqslant w\phi H + rK - I + \pi$

where $\pi = Y - w\phi H - rK$

$\Rightarrow C \leqslant w\phi H + rK - I + (Y - w\phi H - rK) = Y - I$

$\Rightarrow C' = Y'$, no I since this is the last period.

Social planner's problem:

$\max_{C, C', \phi, K', H'} u(C) + u(C')$

s.t. $C \leqslant Y - I$

$C' = Y'$

$H' = H + (1 - \phi) H$

$K' = (1 - \delta) K + I$

$C \leqslant Y - I = K^{\alpha}(\phi H)^{1-\alpha} - (K' - (1 - \delta) K)$

$C' = Y' = K'^{\alpha}(\phi' H')^{1-\alpha} = K'^{\alpha}(\phi' (2 - \phi) H)^{1-\alpha}$

$H' = H + (1 - \phi) H \Rightarrow H' = (2 - \phi) H$

$$K' = (1-\delta)\,K + I \Rightarrow I = K' - (1-\delta)\,K$$

$$\Rightarrow \max_{\phi, K'} u(K^{\alpha}(\phi H)^{1-\alpha} - (K' - (1-\delta)\,K)) + u(K'^{\alpha}(\phi'\,(2-\phi)\,H)^{1-\alpha})$$

$\phi' = 1$ since no third period

$$\Rightarrow \max_{\phi, K'} u(K^{\alpha}(\phi H)^{1-\alpha} - (K' - (1-\delta)\,K)) + u(K'^{\alpha}((2-\phi)\,H)^{1-\alpha})$$

$$[K']: u'(C) = u'(C')\,\alpha\,K'^{\alpha-1}((2-\phi)\,H)^{1-\alpha}$$

$$\Rightarrow \frac{u'(C)}{u'(C')} = \alpha\,K'^{\alpha-1}((2-\phi)\,H)^{1-\alpha}$$

$$[\phi]: u'(C)\,(1-\alpha)K^{\alpha}(\phi H)^{-\alpha}\,H = u'(C')\,K'^{\alpha}\,(1-\alpha)\,((2-\phi)\,H)^{-\alpha}\,H$$

$$\Rightarrow u'(C)K^{\alpha}(\phi H)^{-\alpha}\,H = u'(C')\,K'^{\alpha}\,((2-\phi)\,H)^{-\alpha}\,H$$

$$\Rightarrow u'(C)K^{\alpha}(\phi)^{-\alpha} = u'(C')\,K'^{\alpha}\,((2-\phi))^{-\alpha}$$

$$\Rightarrow \frac{u'(C)}{u'(C')} = \left(\frac{K'}{K}\right)^{\alpha}\left(\frac{(2-\phi)}{\phi}\right)^{-\alpha}$$

$$\Rightarrow \left(\frac{K'}{K}\right)^{\alpha}\left(\frac{(2-\phi)}{\phi}\right)^{-\alpha} = \alpha\,K'^{\alpha-1}((2-\phi)\,H)^{1-\alpha}$$

$$\Rightarrow K'\left(\frac{1}{K}\right)^{\alpha}\left(\frac{1}{\phi}\right)^{-\alpha} = \alpha\,(2-\phi)\,H^{1-\alpha}$$

$$\Rightarrow K'\,K^{-\alpha}\,\phi^{\alpha} = \alpha\,(2-\phi)\,H^{1-\alpha}$$

$$\Rightarrow \frac{\phi^{\alpha}}{2-\phi}\,K' = \alpha\,K^{\alpha}\,H^{1-\alpha}$$