Lecture 11, distorted tax with Cobb-Douglas Production function

Production Function: $Y = zN^a$

Rogerson (1988) utility function: $U(C, N) = \ln C - bN$

Budget constraint: $C = w(1-t)N + \pi$ and $N + l = h = 1$

$D_N U = -b; D_C U = \frac{1}{C} \Rightarrow \text{MRS}_{N,C} = \frac{D_N U}{D_C U} = \frac{-b}{1/C} = -bC$

$\because l = 1 - N$, $\therefore \text{MRS}_{l,C} = -\text{MRS}_{N,C} = bC$

Firm's Problem: $\max_N \pi = zN^a - wN$

FOC: $\frac{d\pi}{dN} = 0 \Rightarrow azN^{a-1} - w = 0 \Rightarrow w(N) = azN^{a-1}$

$\pi(N) = zN^a - w(N)\, N = zN^a - azN^{a-1}N$

$\Rightarrow \pi(N) = zN^a - azN^a = (1-a)\, zN^a$

So, this is the property of Cobb-Douglas production function:

$(1-a)$ fraction of the total output goes to the firm's profit, and a fraction of the total output goes to the labor income for household, i.e., $wN = azN^a$.

In equilibrium, the $\text{MRS}_{l,C}$ will equal to the after-tax wage, i.e.,

$$\text{MRS}_{l,C} = w(1-t) \tag{1}$$

$$\begin{aligned}
\text{MRS}_{l,C} &= w(1-t) \\
bC &= w(1-t) \\
\text{sub. } C \quad & b[w(1-t)N + \pi] \\
&= w(1-t) \\
\text{sub. } w \quad & b[(azN^{a-1})(1-t)N + \pi] \\
&= (azN^{a-1})(1-t) \\
\text{sub. } \pi \quad & b[(azN^{a-1})(1-t)N + (1-a)\, zN^a] \\
&= azN^{a-1}(1-t) \\
& b[azN^a(1-t) + (1-a)\, zN^a] \\
&= azN^{a-1}(1-t) \\
\text{factor } zN^a \quad & zN^a\, b[a\,(1-t) + (1-a)] \\
&= azN^{a-1}(1-t)
\end{aligned}$$

$$
\begin{aligned}
\text{divide } zN^{a-1} \qquad & Nb[a(1-t)+(1-a)] \\
&= a(1-t) \\
N &= \frac{a(1-t)}{b[a(1-t)+(1-a)]} \\
w(N) &= az\left(\frac{a(1-t)}{b[a(1-t)+(1-a)]}\right)^{a-1} \\
\pi(N) &= (1-a)z\left(\frac{a(1-t)}{b[a(1-t)+(1-a)]}\right)^{a}
\end{aligned}
$$

Government's objective can be

1. fulfill the exogenous government spending: $G = R(t) = wtN$

2. maximize the tax revenue by choosing a tax rate: $\max_t R(t) = w(t)\,t\,N(t)$.