

Lecture 5

Representative Consumer Optimization and Application

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Overview: Lecture 4 - 7

Provide **micro-foundation** for the **macro implication** (Lucas critique)

■ Representative Consumer:

- Lecture 4: **preference, constraints**
- Lecture 5: **optimization, application**
- Lecture 6: Numerical Examples

■ Representative Firm:

- Lecture 7: **production, optimization, application**

Review: MRS

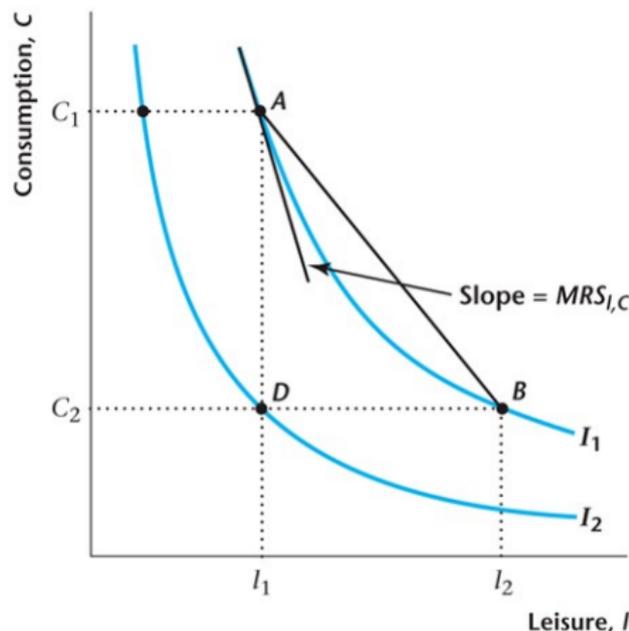
■ Normality: Marginal Rate of Substitution

- **Marginal:** for arbitrary small change in x -axis (leisure in this case)
- **rate of substitution:** the amount on y -axis has to be sacrificed (consumption in this case)

$$MRS_{l,C} = \frac{D_l U(C, l)}{D_C U(C, l)}, \quad (1)$$

where $D_x U(\cdot)$ is derivative of U w.r.t. x

Figure 4.2 MRS



Consumer's Problem

The consumer choose **consumption** and **leisure** bundle to achieve **highest** indifference curve, while still satisfying **budget constraint**

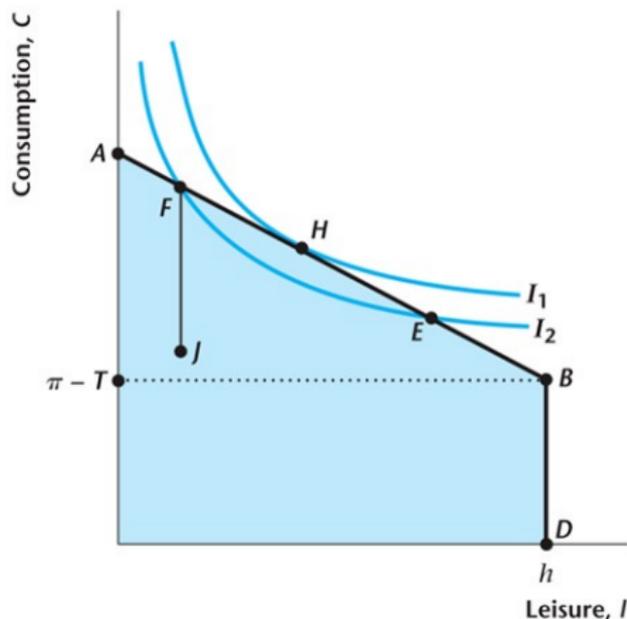
$$\begin{aligned} \max_{C,l} \quad & U(C, l) \\ \text{subject to} \quad & C \leq w(h - l) + \pi - T \end{aligned} \tag{2}$$

- **Rational behavior**: decision is made given preference & constraints
- **Analysis**: both **graphically** and **algebraically**

Graphical Analysis: Interior Solution

- **Interior:** sol. at middle of budget set, not end pts
- **MRS** must equal to **real wage** ($MRS_{l,C} = w$), **WHY?**
 - sacrificed consumption comes from the decrease of labor income
- Sol. at indifference curve **tangent** to budget set
- **Convexity:** E v.s. H & F v.s. H

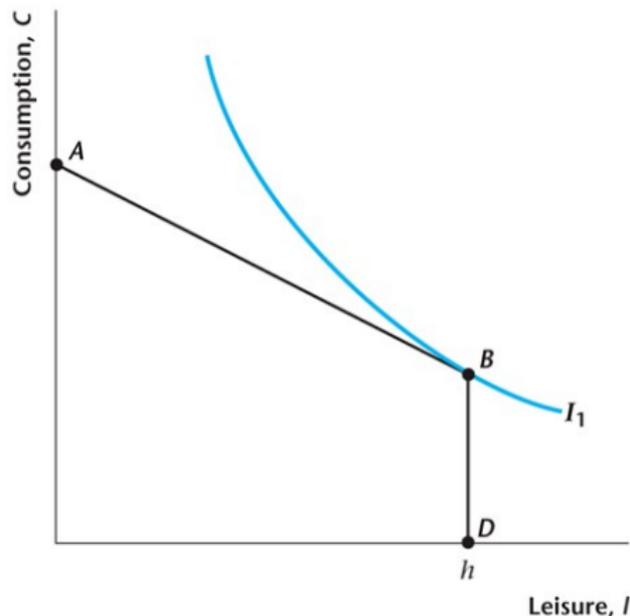
Figure 4.5 Interior Solution



Graphical Analysis: Corner Solution

- **Corner:** sol. at end pts of budget set
- **MRS NOT** equal to **real wage** ($MRS_{l,C} \neq w$), WHY?
 - working limited to total h hours, “kink”
- Sol. is NOT tangent to indifference curve

Figure 4.6 Corner Solution



Algebraic Analysis: Interior Solution

Recall consumer's problem:

$$\begin{aligned} \max_{C,l} \quad & U(C, l) \\ \text{subject to} \quad & C \leq w(h - l) + \pi - T \end{aligned} \tag{3}$$

- Calculus is about **derivative**: not defined at “kink” \Rightarrow only **interior sol.**
- Sol. at the **border** of budget set \Rightarrow budget constraint is “=” (**binding**)

Plug the budget constraint into utility function to replace C , we get

$$\max_l \quad U(w(h - l) + \pi - T, l) \tag{4}$$

Algebraic Analysis: Interior Solution (Cont.)

$$\max_l U(w(h-l) + \pi - T, l)$$

Remember that now $C = w(h-l) + \pi - T$. Take **first order condition** w.r.t. l ,

$$\overbrace{D_C U(C, l) \times \frac{d[w(h-l) + \pi - T]}{dl}}^{\text{Derivative on } C \text{ direction, chain rule}} + \overbrace{D_l U(C, l)}^{\text{Derivative on } l \text{ direction}} = 0 \quad (5)$$

$$D_C U(C, l) \times (-w) + D_l U(C, l) = 0 \quad (6)$$

$$w = \frac{D_l U(C, l)}{D_C U(C, l)} = MRS_{l,C} \quad (7)$$

Note: $D_x f(\cdot)$ is a shorthand for $\frac{df(\cdot)}{dx}$, meaning **differentiation** of $f(\cdot)$ with respect to choice variable x .

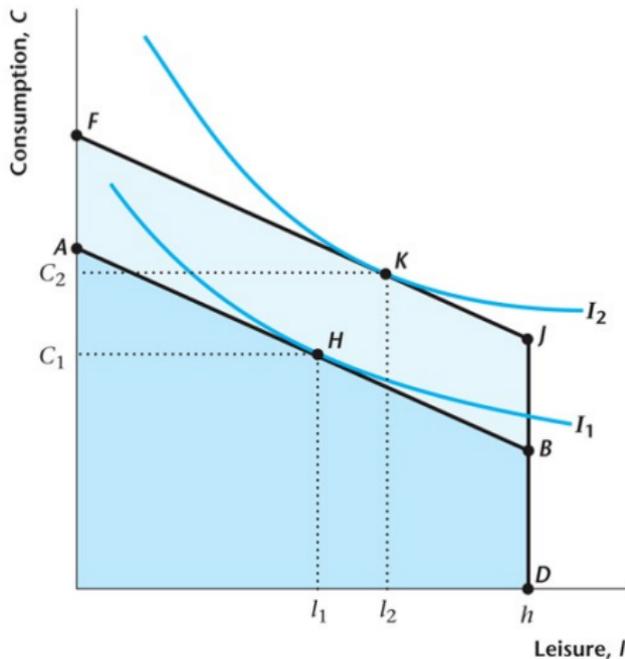
Build model for experiment

- We want to know **what's the result of changes!**
- Recall **Lucas critique**: need to understand individual behavior
- Consider two experiments:
 - ① direct increase in **real income** (no C and l trade off, pure **income effect**)
 - ② increase in **real wage** (**income + substitution effect**)

Experiment 1: Increase in dividends / Decrease in Tax

- **Recall:** C & l are normal goods
- **Income effect:** income $\uparrow \Rightarrow$ normal goods \uparrow
- Increase in dividends or decrease in taxes are level shifts up in real income, regardless of actions
- Consumer increases consumption, reduces quantity of labor supplied (increase leisure).

Figure 4.6 $\pi \uparrow / T \downarrow$



Experiment 2: Increase in Real Wage

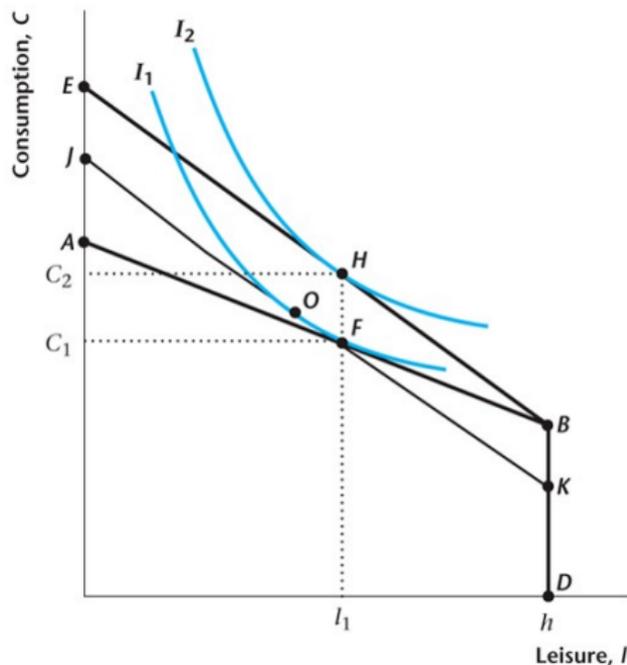
Substitution effect: $w \uparrow$, leisure is costly, sacrifice l for C

- budget line AB to JK, keeps F just affordable
- move along I_1 : new slope of budget line

Income effect: income $\uparrow \Rightarrow$ normal goods \uparrow

- budget line JK to EB, actual new budget line
- move up to I_2 : higher utility possible

Figure 4.8 $w \uparrow$, both effects canceled out

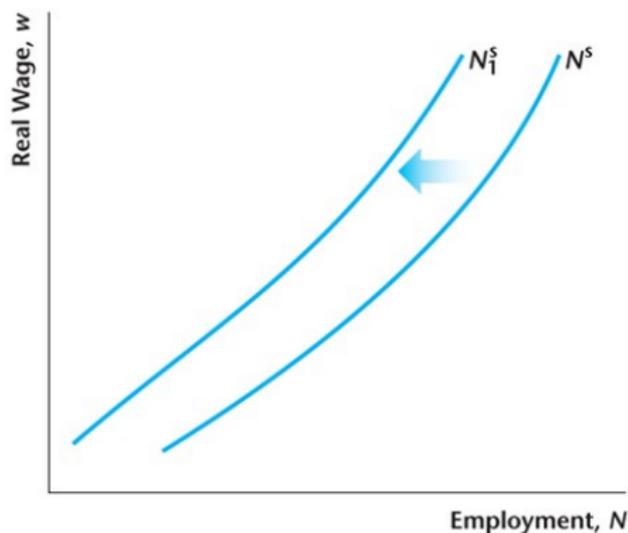


Experiment 1 & 2: Labor Supply

Looking ahead to putting the pieces together in a full model:

- Solution to consumer problem defines the supply curve for the labor market!
- What assumption ensures this is increasing in the wage?
 - substitution effect:
 $w \uparrow \Rightarrow l \downarrow$ (i.e., $N^S \uparrow$)
 - income effect:
 $w \uparrow \Rightarrow l \uparrow$ (i.e., $N^S \downarrow$)
 - Income effect < substitution effect

Figure 4.10, LS on $\pi \uparrow / T \downarrow$



Appendix

Chain rule

Back

In the main slide, we applied **chain rule** to the C direction of the $U(C, l)$. By binding budget constraints, we know $C(l) = w(h - l) + \pi - T$, i.e., consumption is a function of leisure.

$$\frac{d}{dl}U(C(l)) = \frac{dU(C, l)}{dC} \times \frac{dC(l)}{dl} = D_C U(C, l) \times D_l C(l) \quad (8)$$

where $D_l C(l) = -w$.