Lecture 8
Competitive Equilibrium
One-Period Model

Hui-Jun Chen

The Ohio State University

June 2, 2022

Overview

After constructing both consumers' and firms' problem, we start to bring them together in one-period model:

- Lecture 8: competitive equilibrium (CE)
  - each agent solve their problems individually
  - aggregate decision determines "prices" (wage, rent, etc.)
- Lecture 9: social planer's problem (SPP)
  - imaginary and benevolent social planner determines the allocation
  - should be the most efficient outcome
- Lecture 10: CE and SPP examples

Review: Structure of Macro Model: 4 elements

1. **agent**: who is involved?
   - e.g. consumers, firms, government
2. **preferences**: how and what is consumed/valued/invested?
   - consumers: monotone, convex, consumption + leisure normal
   - firms: profit maximization
   - government: passive (for now)
3. **resources**: availability and distribution
   - consumer: h unit of time endowment
   - firm: production technology $zF(K, N^d)$
4. **technology**: objective limitation at given period of time
   - CRS production function, government tax decision

Government and Budget Balance

Government provide G unit of gov. spending by imposing lump-sum tax T to representative consumer.

Assumptions:

1. Gov. spending requires resources but with no benefit
   - not public goods
2. no transfers between consumers
3. **gov. budget balance**: $G = T$, must run balanced budget
   - special case: $G = 0$ means no government!

Using a Macro Model

"Making use of the model is a process of running experiments to determine how changes in the exogenous variables change the endogenous variables." – Williamson, p.144

Exogenous variables: determined outside the model

1. G: gov. spending
2. K: firms' capital stock
3. z, h: TFP, consumer's time endowment

Endogenous variables: determined inside the model

- C, Y: consumption, output
- N^s, N^d: labor supply & demand
- T, w, π: tax level, wage rate, dividends

Concept: Competitive Equilibrium

- Agents in the economy behave for a given set of exogenous variables and parameters
- Both consumer and firm took the wage rate as given.
- But this wage is endogenous! How is this wage determined?
- Solution: in competitive equilibrium,
  - prices are exogenous to agent ("taken as given"), but
  - endogenous to the model (NOT parameter and need to be solved)
- **Market clear**: wage rate is determined by $N^s = N^d$ ("endogenous")
- other examples: dividend income, taxes

Analysis on Competitive Equilibrium

- How many markets exist in this economy?
  - There are 2 goods: consumption goods and leisure
  - While there is only 1 market: leisure is traded for consumption with wage rate w
- **Walras' Law**: with N goods, can only have $N-1$ prices
  - All prices are relative prices:
    - normalize price of consumption as 1, the relative price of leisure is w
  - Trade consumption goods for consumption goods?

Competitive Equilibrium in Words

A competitive equilibrium given *exogenous levels of government spending, TFP, and capital* is a set of endogenous quantities of output, consumption, labor demand, labor supply, dividends, and taxes and an endogenous wage rate such that the following properties are satisfied:

1. the representative consumer chooses consumption and labor supply to make herself as well off as possible subject to her budget constraint, taking as given the wage, taxes, and dividend income
2. the representative firm chooses labor demand to maximize profits taking capital, TFP, and the wage as given.
3. output (profits) are total (net) revenues, determined "residually"
4. the government imposes the taxes required by its budget constraint
5. the labor market clears, i.e., the quantity of labor supplied by the consumer is equal to the quantity of labor demanded by the firm.

Competitive Equilibrium in Math

A competitive equilibrium given $\{G, z, K\}$ is a set of allocations $\{Y^*, C^*, l^*, N^{s*}, N^{d*}, \pi^*, T^*\}$ and prices $\{w^*\}$ such that

1. Taken prices w and π, T as given, representative consumer solves

$$\max_{C, l \in [0,h]} U(C, l) \quad \text{subject to} \quad C \leq w(h-l) + \pi - T \tag{1}$$

2. Taken w as given, the representative firm solves

$$\max_{N^d \geq 0} zF(K, N^d) - wN^d \tag{2}$$

3. Government set taxes to balance budget: $T^* = G$

4. Labor market clears: w^* such that $N^{s*} = N^{d*}$

Does it All Add Up?

Revisiting the Income-Expenditure Identity

- **Expenditure approach**: $Y = C + I + G + NX$
  - one period $\Rightarrow I = 0$; closed economy $\Rightarrow NX = 0 \Rightarrow Y = C + G$
- **Income approach**:
  - consumer budget constraint: $C = wN^s + \pi - T$
  - government budget balance: $G = T \Rightarrow C = wN^s + \pi - G$
  - profit:
    $\pi = zF(K, N^d) - wN^d = Y - wN^d \Rightarrow C = wN^s + Y - wN^d - G$
  - labor market clear: $N^s = N^d \Rightarrow C = Y - G$
- **Income-Expenditure Identity holds!**

Example

Assume

1. no government: $G = T = 0$
2. utility function: $U(C, l) = \ln C + \ln l$
3. production function: $F(K, N) = K^{\alpha} N^{1-\alpha}$, where $\alpha = \frac{1}{2}$
4. $z = K = 1$; $h = 1$

Consumer: $\max_{C,l} \ln C + \ln l$ subject to $C \leq w(h - l) + \pi$

$$\text{FOC} \quad \frac{C}{l} = w \tag{3}$$

$$\text{Binding budget constraint} \quad C = w(1 - l) + \pi \tag{4}$$

$$\text{Time constraint} \quad N^s = 1 - l \tag{5}$$

Example (Cont.)

Firm: $\max_{N^d}(N^d)^{\frac{1}{2}} - wN^d$

$$\text{FOC} \quad \frac{1}{2}(N^d)^{-\frac{1}{2}} = w \tag{6}$$

$$\text{Output definition} \quad Y = (N^d)^{\frac{1}{2}} \tag{7}$$

$$\text{Profit definition} \quad \pi = Y - wN^d \tag{8}$$

Market clear:

$$N^s = N^d \tag{9}$$

7 equations ((3)-(9)), 7 unknowns $(C, l, N^s, N^d, Y, \pi, w)$, can solve entirely!