Lecture 9
Social Planner's Problem

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Overview

After constructing both consumers' and firms' problem, we start to bring them together in one-period model:

- Lecture 8: competitive equilibrium (CE)
  - each agent solve their problems individually
  - aggregate decision determines "prices" (wage, rent, etc.)
- Lecture 9: social planer's problem (SPP)
  - imaginary and benevolent social planner determines the allocation
  - should be the most efficient outcome
- Lecture 10: CE and SPP examples

What is Social Planner?

- Benevolent dictator whose goal is to maximize social welfare given technological constraint
- **Social welfare**: joint "happiness" of every agent in this economy
  - consumer: tangency between IC and budget line in (C, l)-plane
  - firm: $Y = zF(K, N) = zF(K, h - l)$
    - labor market clearing: $N = N^s = N^d$
    - consistent with consumer behavior: $N = h - l$
  - government: income-expenditure identity, $C = Y - G$
    - government is not necessary the social planner! (also one of the agents)
- **Technological constraint**: production possibility frontier

Production Possibility Frontier (PPF)

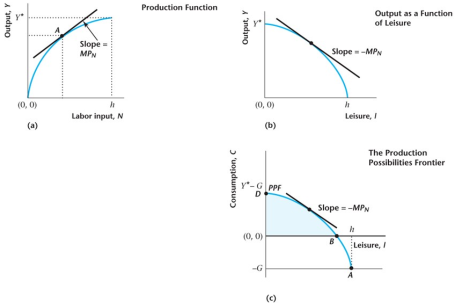
- **Def**: technological possibilities for the whole economy

$$C = zF(K, h - l) - G \quad (1)$$

- **Marginal rate of transformation** (MRT): rate to transform leisure to consumption (through work)

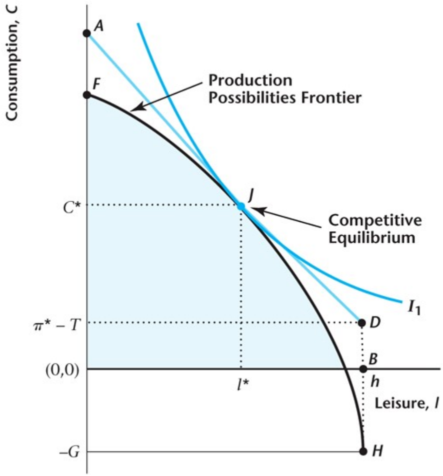
$$\begin{aligned} MRT_{l,C} &= zD_N F(K, N) \\ &= MPN \end{aligned} \quad (2)$$

Figure 5.2 The Production Function and the Production Possibilities Frontier



Competitive Equilibrium: Graphcial Representation

Figure 5.3 Competitive Equilibrium



Combine PPF with IC:

- $\overline{AD}$: tangent to consumer's IC I_1 and PPF $\overline{FH}$
- negative slope of $\overline{AD}$: equilibrium wage w
  - $\because \overline{AD}$ is budget line
- Recall Lecture 8 & last slide:
  - conumser: $MRS_{l,C} = w$
  - firm: $MPN = w$
  - efficiency: $MRT_{l,C} = MPN$

$$MRS_{l,C} = MRT_{l,C} = MPN$$

Concept: Pareto Improvement / Optimal

A competitive equilibrium is **Pareto optimal** or **Pareto efficient** if there is no way to rearrange production or to reallocate goods so that someone is made better off without making someone else worse off.

- only one consumer, so relatively straightforward
- but, still a powerful concept:
  - free markets can produce socially efficient outcomes
  - often easier to analyze social optimum than competitive equilibrium
- caveats:
  - "efficiency" in economics is a statement about a model
  - very narrow: e.g. having Jeff Bezos pay for a meal for someone in need.

Social Planner's Problem

objective: consumer's utility $\max_{C,l,N,Y} U(C,l)$

subject to

agg. resource constraint $C + G \leq Y$

production constraint $Y = zF(K,N)$

labor constraint $N = h - l$

- What's here: GDP accounting, physical / technological constraints, required government spending, consumer preferences
- What's not: consumer's budget constraint, the wage rate, consumer's / firm's individual problems, profits, taxes

Solving Social Planner's Problem

We know all constraints bind, so by substituting:

$$\max_{l} U(zF(K, h-l) - G, l) \tag{3}$$

FOC:

$$\begin{aligned} &D_l U(zF(K, h-l) - G, l) \\ =&D_C U(zF(K, h-l) - G, l)(zD_N F(K, h-l)) \end{aligned} \tag{4}$$

Rearrange:

$$\frac{D_l U(zF(K, h-l) - G, l)}{D_C U(zF(K, h-l) - G, l)} = zD_N F(K, h-l) \Rightarrow MRS_{l,C} = MRT_{l,C} \tag{5}$$

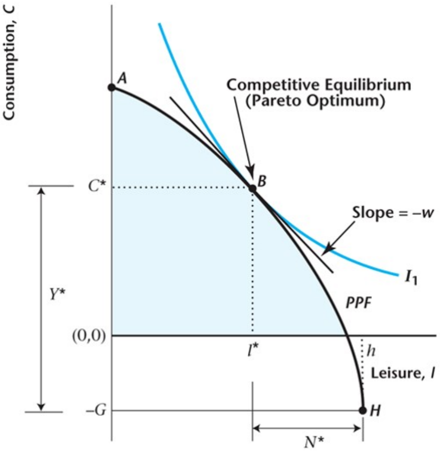
Same Result! Why?

Welfare Theorem

- **First welfare theorem**: under centain conditions, the allocation under a competitive equilibrium is Pareto optimal
- **Second welfare theorem**: under certain conditions, a Pareto optimal allocation is the allocation for a competitive equilibrium.
- straightforward to show here (we already have!) but no always so.
  - conditions not always met!
- SPP and CE often alike if not identical, serves as a good benchmark

Social Planner's Problem: Graphical Representation

Figure 5.4 Pareto Optimality



Apply SPP & 2nd welfare theorme for competitive equilibrium:

- l^* determined by SPP at B
- C^*, N^*, Y^* by plugging into constraints
- $w^* = MPN = MRT_{l,C} = MRS_{l,C}$

What Can Go Wrong? Cases when SPP $\neq$ CE

1. Externalities: activity for which an individual does not take account of all associated costs and benefits: can be positive or negative
   - example: pollution must be cleaned up, but firm doesn't have to
2. Distorting taxes: lead to "wedges" between MRS, MP, and MRT
   - example: proportional labor income tax vs lump-sum tax
3. Non-competitive / monopolistic behavior: firms or consumers may not be price takers
   - examples: local media markets, negotiations