

Lecture 14
The Real Business Cycle Model
Part 1: Consumer

Hui-Jun Chen

The Ohio State University

July 7, 2022

Overview

- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
 - Lecture 14: consumer
 - Lecture 15: firm
 - Lecture 16: competitive equilibrium
 - Lecture 17: formal example
 - Lecture 18: application to bring RBC to data

Real Business Cycle Model

- One of the workhorse frameworks in modern macroeconomics
- **Real**: not about *money* and *inflation*
- **Business Cycle**: mainly explain the short- and medium-term economics fluctuation (“business cycle frequency”)
- Three agents: representative consumer, representative firm, and government
- All agents make **static and dynamic** decisions
- Larger “scale” model (i.e., more endogenous variables), but build upon the technique learned before

Consumer: Constraints

There are 11 variables associated with the representative consumer:

- choice variables: consumption (C, C') and labor supply (N_S, N'_S)
 - leisure follows labor choice: $l = h - N_S$, and $l' = h - N'_S$
- owns the firm and get profits (π, π') and pays taxes (T, T')
- taken the equilibrium price as given (w, w', r)

Saving (S) at date 0 to construct lifetime budget constraint:

$$\text{today: } C + S = wN_S + \pi - T$$

$$\text{tomorrow: } C' = w'N'_S + \pi' - T' + (1+r)S$$

$$\text{lifetime constraint: } C + \frac{C'}{1+r} = \underbrace{wN_S + \pi - T}_{\approx Y \text{ in last lecture}} + \frac{\overbrace{w'N'_S + \pi' - T'}^{\approx Y' \text{ in last lecture}}}{1+r}$$

Consumer: Preference

In general, utility fcn across consumption and labor choice can be mixed:

- e.g. mix C and N_S : **GHH preferences**
- e.g. mix current and future: **Epstein–Zin preferences**

Here, we are making simplified assumption: **additive for both direction**:

$$U(C, C', N_S, N'_S) = u(C) - v(N_S) + u(C') - v(N'_S). \quad (1)$$

To see why **additive** can simplify analysis, recall the MRS in both **intra-temporal** (w/i period) and **inter-temporal** (b/w period) substitution:

$$MRS_{l,C} = -MRS_{N_S,C} = \frac{v'(N_S)}{u'(C)}, \text{ and } MRS_{C,C'} = \frac{u'(C)}{u'(C')}.$$

Representative Consumer's Problem

$$\begin{aligned} & \max_{C, C', N_S, N'_S} && u(C) - v(N_S) + u(C') - v(N'_S) \\ \text{subject to} &&& C + \frac{C'}{1+r} = wN_S + \pi - T + \frac{w'N'_S + \pi' - T'}{1+r} \end{aligned} \quad (2)$$

- Hard to analyze in graph, \because 4 choices variables \Rightarrow 4-dim problem!
- Yet, usual procedure in Calculus still works!
- Why? Because **partial derivatives** only looks the optimality in **1-dim**
- Each FOC is optimal for 1-dim \Rightarrow solution satisfies ALL FOCs

Consumer's Optimality Conditions

- Step 1: substitute C by budget constraint,

$$\begin{aligned} \max_{C', N_S, N'_S} u \left(wN_S + \pi - T + \frac{w'N'_S + \pi' - T' - C'}{1+r} \right) \\ - v(N_S) + u(C') - v(N'_S) \end{aligned}$$

- Step 2: find FOCs for C' , N_S , and N'_S :

$$[C'] : u'(C') - \frac{1}{1+r}u'(C) = 0 \Rightarrow u'(C') = \frac{1}{1+r}u'(C)$$

$$[N_S] : wu'(C) - v'(N_S) = 0 \Rightarrow wu'(C) = v'(N_S)$$

$$[N'_S] : \frac{w'}{1+r}u'(C) - v'(N'_S) = 0 \Rightarrow \frac{w'}{1+r}u'(C) = v'(N'_S)$$

Consumer's Optimality Conditions (Cont.)

- Step 3: Compute multiple MRSs:

$$[C'] : \quad MRS_{C,C'} = \frac{u'(C)}{u'(C')} = 1 + r$$

$$[N_S] : \quad -MRS_{N_S,C} = MRS_{l,C} = \frac{v'(N_S)}{u'(C)} = w$$

$$[N'_S] : \quad MRS_{N'_S,C} = \frac{v'(N'_S)}{u'(C)} = \frac{w'}{1+r}$$

- Step 4: Get C by putting C' , N_S and N'_S back to budget constraint.

Knowledge Gain from Consumer's Problem

We have derived 4 optimality conditions for 4 choice variables:

$$[C'] : MRS_{C,C'} = \frac{u'(C)}{u'(C')} = 1 + r$$

$$[N_S] : -MRS_{N_S,C} = MRS_{l,C} = \frac{v'(N_S)}{u'(C)} = w$$

$$[N'_S] : MRS_{l',C} = \frac{v'(N'_S)}{u'(C)} = \frac{w'}{1+r}$$

$$\text{budget constraint : } C = wN_S + \pi - T + \frac{w'N'_S + \pi' - T' - C'}{1+r}$$

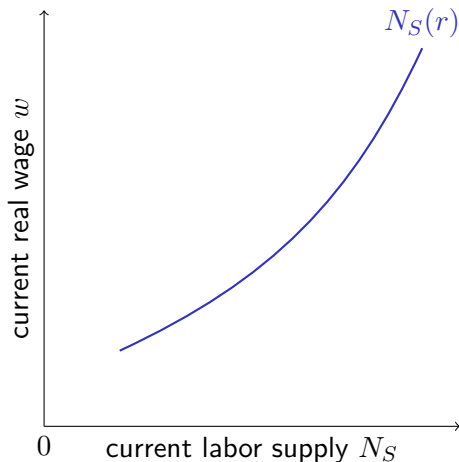
Recall that there are 11 variables, so still 7 variables remain. They are:

- 3 endogenous prices: w, w', r
- 4 endogenous quantities that shift lifetime wealth: π, π', T, T'

Need to know how consumer response to endogenous quantities!

Current Labor Supply and Current Wage

Figure 11.1 The Representative Consumer's Current Labor Supply Curve

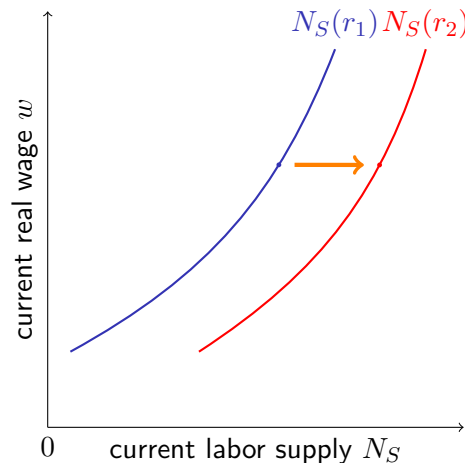


Assumption N1: current labor supply \uparrow in current wage

- Recall two effects of wage on labor:
 - income (I): $l \uparrow, N_S \downarrow$
 - substitution (S): $l \downarrow, N_S \uparrow$
- **N1** suggests that **substitution effect** $>$ **income effect**
- data: (I) and (S) cancel out in long-run, while RBC focus on **short- and medium run!**

Current Labor Supply and Real Interest Rate

Figure 11.2 Real Interest Rate \uparrow Shifts the Current Labor Supply Curve to the Right



Assumption N2: current labor supply \uparrow as real interest rate \uparrow

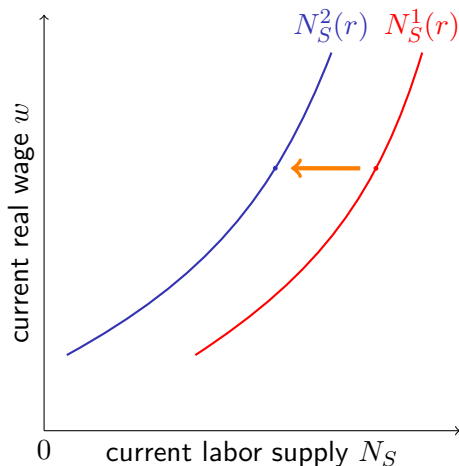
- can substitute **intertemporally** using **both** consumption and labor
- relative price of future leisure in terms of current leisure:

$$\begin{aligned} \frac{v'(N_S)}{v'(N'_S)} &= \frac{v'(N_S)}{u'(C)} \times \frac{u'(C)}{u'(C')} \times \frac{u'(C')}{v'(N'_S)} \\ &= w \times (1 + r) \times \frac{1}{w'} \end{aligned}$$

- fix w and w' , $r \uparrow$ makes l become more costly, so $N_S \uparrow$

Current Labor Supply and Wealth

Figure 11.3 Effects of an Increase in Lifetime Wealth



Assumption N3: current labor supply \downarrow as lifetime wealth \uparrow

- only pure income effect on normal goods (consumption & leisure), and thus labor decreases

Summary of Effect on Labor Supply

- **Assumption N1:** current labor supply \uparrow in current wage
 - $\frac{dN_S}{dw} > 0$
- **Assumption N2:** current labor supply \uparrow as real interest rate \uparrow
 - $\frac{dN_S}{dr} > 0$
- **Assumption N3:** current labor supply \downarrow as lifetime wealth \uparrow
 - $\frac{dN_S}{dx} < 0$, where $x = \pi - T$.

All statements are properties about **supply curve**, not equilibrium quantities!