Lecture 13
Competitive Equilibrium in Two-Period Model

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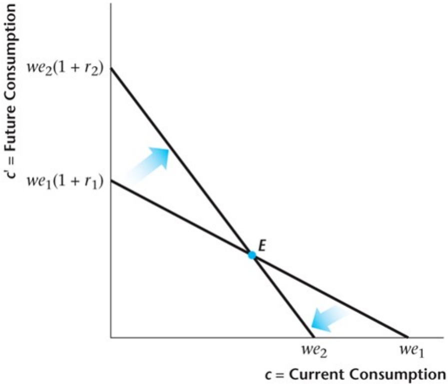
Increase in Real Interest Rate

real interest rate r increase $\Rightarrow$ budget line rotate

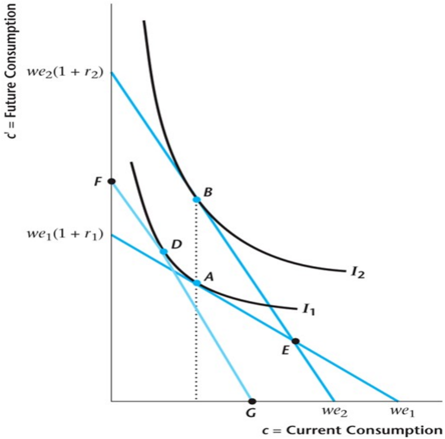
Figure 9.12 An Increase in the Real Interest Rate



- Recall $we = y - t + \frac{y' - t'}{1 + r}$, $r \uparrow \Rightarrow we \downarrow$
- can do nothing: pivot around E
- similar to wage increase (slope $\uparrow$)
- income & substitution effects (change in relative price)
- income effect depends on the sign of saving s

Increase in Real Interest Rate: Effect on Lender ($s > 0$)

Figure 9.13 An Increase in the Real Interest Rate for a Lender

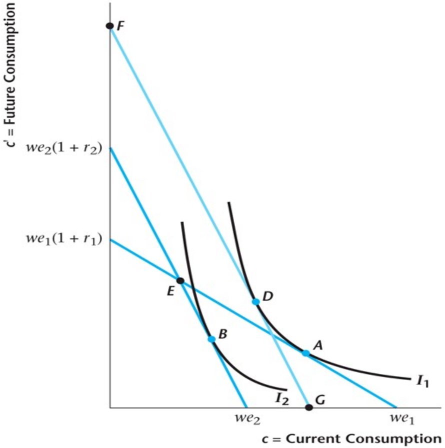


Let initial bundle be A.

- **Substitution effect**: rotate from $\overline{AE}$ to $\overline{FG}$
  - $\because r\uparrow$, current consumption become more expensive $\Rightarrow$ $c_D < c_A, c'_D > c'_A$
- **Income effect**: shift from $\overline{FG}$ to $\overline{BE}$
  - normality: $c_B > c_D, c'_B > c'_D$
  - $c' \uparrow$, $\because$ both effects aligned
  - c and $s = y - t - c$ are ambiguous, $\because$ both effects contradict

Increase in Real Interest Rate: Effect on Borrower ($s < 0$)

Figure 9.14 An Increase in the Real Interest Rate for a Borrower



Let initial bundle be A.

- **Substitution effect**: rotate from $\overline{AE}$ to $\overline{FG}$
  - $\because r \uparrow$, current consumption become more expensive $\Rightarrow$ $c_D < c_A, c'_D > c'_A$ [same as lender!]
- **Income effect**: shift from $\overline{FG}$ to $\overline{BE}$
  - normality: $c_B < c_D, c'_B < c'_D$ [opposite to lender!]
  - $c, s \downarrow$, $\because$ both effects aligned
  - c' is ambiguous, $\because$ both effects contradict

Summary

Both borrowers and lenders experience intertemporal substitution:

- $r \uparrow \Rightarrow$ cost of current consumption $\uparrow \Rightarrow c \downarrow$
- aggregate effect depends on the distribution of borrowers and lenders
  - $\because$ both effects are in opposite directions
  - important and active research topic in macro!
- tendency for confounding income effects on borrowers and lenders to roughly cancel out, still effect on aggregate consumption is not guaranteed.

Government in Two-Period Model

Impose lump-sum tax T and issue government bond B to finance government spending G in each period.

- government purchase G unit of good today and G' tomorrow,
- impose T and T' of lump-sum taxes to consumers, and
- Issue B unit of bond today and pay back $(1+r)B$ tomorrow.

Budget constraints:

$$\text{date 0}: \quad G = T + B \tag{1}$$

$$\text{date 1}: \quad G' + (1+r)B = T' \tag{2}$$

$$\Rightarrow \text{lifetime budget constraint}: \quad G + \frac{G'}{1+r} = T + \frac{T'}{1+r} \tag{3}$$

Budget deficit is allowed in one period, but **must be repaid** in the future.

Two-Period Competitive Equilibrium in Words

A competitive equilibrium given government spending and consumers' endowment is a set of **endogenous quantities and prices** of current and future consumption, current and future lump-sum taxes, savings, government bond, as well as the real interest rate such that

1. Taken the real interest rate and lump-sum taxes as given, **consumers** maximized their lifetime utility subject to the intertemporal budget constraints.
2. Taken the real interest rate as given, the intertemporal **government** budget constraint holds.
3. The credit market clears determines the equilibrium real interest rate.

Two-Period Competitive Equilibrium in Math

A competitive equilibrium given exogenous quantities $\{G, G', Y, Y'\}$, is a set of **endogenous quantities and prices** $\{C, C', S, T, T', B, r\}$

1. Taken r, T, and T', **consumers** solve

$$\max_{C,C'} U(C, C') \quad \text{subject to} \quad C + \frac{C'}{1+r} = Y - T + \frac{Y' - T'}{1+r},$$

where solutions are C^*, C'^*, and $S^* = Y - T - C^*$.

2. The present value of **government budget constraint** holds:

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r},$$

where government bond B is determined by $B = G - T$.

3. The **credit market clears**: $S = B$ at the equilibrium interest rate r^*.

The Credit Market and GDP Accounting

In one-period model, firm and consumer interact in the labor market.

Here, government and consumer interact in the credit market.

- S is private saving, and $-B = S^g$ is public saving
- closed economy: national net saving must equals 0, so $S - B = 0$.

$$
\begin{aligned}
\text{current consumer budget:} \quad & S = Y - T - C \\
\text{with current gov budget:} \quad & S = Y - (G - B) - C \\
S = B: \quad & Y = C + G \\
\text{future consumer budget:} \quad & (1+r)S = C' + T' - Y' \\
\text{with future gov budget:} \quad & (1+r)S = C' + (G' + (1+r)B) - Y' \\
S = B: \quad & Y' = C' + G'
\end{aligned}
$$

An Example

Suppose $G = G' = T = T' = B = 0$, i.e., government is ignored, then

- **consumer**: let $U(C, C') = \ln C + \ln C'$, and $Y = Y' = 1$,

$$\max_{C,C'} \ln C + \ln C' \quad \text{subject to} \quad C + \frac{C'}{1+r} = 1 + \frac{1}{1+r}$$

- FOC:

$$MRS_{C,C'} = \frac{C'}{C} = 1 + r \quad \Rightarrow \quad C + \frac{(1+r)C}{1+r} = \frac{2+r}{1+r}$$
$$\Rightarrow \quad 2C = \frac{2+r}{1+r} \Rightarrow C^* = \frac{2+r}{2(1+r)}$$

- **credit market clear:**

$$S = B = Y - T - C^* = 1 - 0 - \frac{2+r}{2(1+r)} = 0 \Rightarrow r^* = 0 \Rightarrow C = C' = 1$$

Ricardian Equivalence

In this model, the timing of taxes is **neutral**: no effect on the real interest rate or on the consumption of individual consumers.

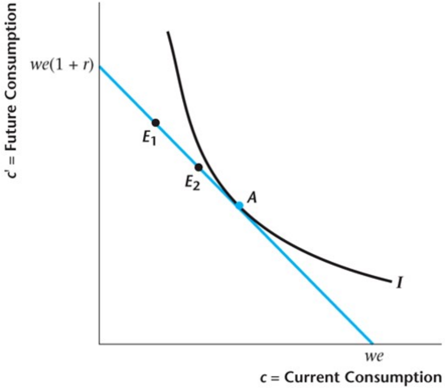
Recall consumer and government budget constraint:

$$
\begin{aligned}
\text{government}: \quad & G + \frac{G'}{1+r} = T + \frac{T'}{1+r} \\
\text{consumer}: \quad & C + \frac{C'}{1+r} = Y + \frac{Y'}{1+r} - \left(T + \frac{T'}{1+r}\right) \\
& \qquad\qquad = Y + \frac{Y'}{1+r} - \left(G + \frac{G'}{1+r}\right)
\end{aligned}
$$

Therefore, for any tax scheme such that government budget constraint holds, there's no effect on r, C and C'.

Ricardian Equivalence in Graph

Figure 9.16 Ricardian Equivalence with a Cut in Current Taxes for a Borrower



Suppose under tax scheme (T, T'), consumer:

- has endowment point E_1
- chooses optimal bundle A

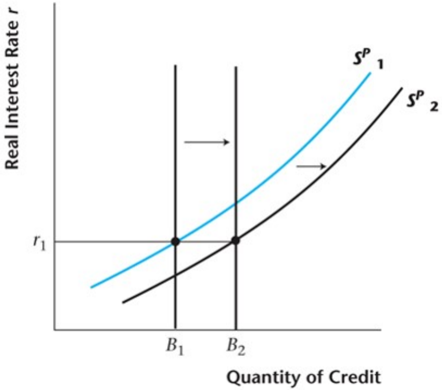
If there's a tax cut scheme $(\tilde{T}, \tilde{T}')$ such that (G, G') remain the same,

- lower current taxes $(\tilde{T} < T)$
- but higher future taxes $(\tilde{T}' > T')$

Then consumer has endowment E_2, but still choose optimal bundle A.

Ricardian Equivalence and Credit Market

Figure 9.17 Ricardian Equivalence and Credit Market Equilibrium



Following the tax cut in last slide,

- $T \downarrow \Rightarrow$ larger deficit today
- Recall $B = G - T$, $B \uparrow$, more bonds today (demand $\uparrow$)
- Recall $S = Y - T - C$, $S \uparrow$, more private saving today (supply $\uparrow$)
- Ricardian Equivalence: both shifts exactly offsets, $r_2 = r_1$
- Recall PIH: tax cut is 100% temporary!

When Will Ricardian Equivalence fail?

This is an extreme result! It provides a useful benchmark to consider richer settings. What can change to "undo" this result?

1. **distribution of tax burden:** consider a case of this model with N consumers, labeled $i = 1, \dots N$. Assume that $T = \sum_{i=1}^{N} t_i$, and consumer i pays t_i.
   - Everyone pays different t_i! What if tax cut not apply to everyone?
2. **consumer lives the whole time:** government can "kick the can" until long in the future, when current generation is retired or dead.
   - redistribution of wealth across generations, social security
3. **distorting taxes:** lump sum not feasible, but proportional distort
4. **imperfect credit market:** borrowing and lending is often "frictional"
   - example: different rates on borrowing and saving, many others!