Review of Mathematics

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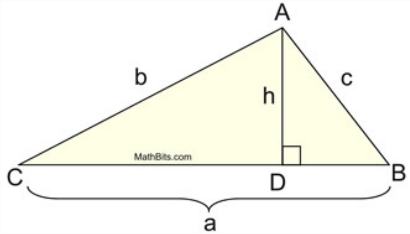
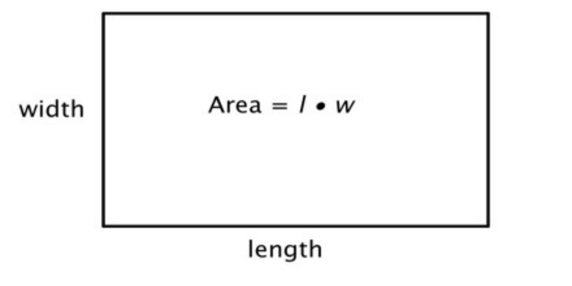
Area Formula

Area Formula: Triangle



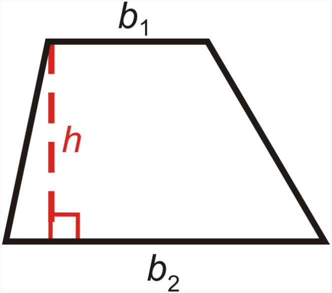
- Area formula: $\frac{1}{2} \times a \times h$

Area Formula: Rectangle



- Area formula: $length \times width$

Area Formula: Trapezoid



- Area formula: $\frac{(b_1+b_2)}{2} \times h$
- Or separate into two triangles and one rectangle

Basic Algebra Review

Basic Algebra Review: properties

- Associative properties:
  - additive: $a + (b + c) = (a + b) + c$
  - multiplicative: $a\,(bc) = (ab)\,c$
- Commutative properties:
  - additive: $a + b = b + a$
  - multiplicative: $ab = ba$
- Distributive properties: $a\,(b + c) = ab + ac$
- Properties for exponents:
  - $a^x a^y = a^{x+y}$; $\frac{a^x}{a^y} = a^{x-y}$
  - $(ab)^x = a^x b^x$; $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
  - $(a^x)^y = a^{xy}$

Basic Algebra Review: properties (Cont.)

- Properties for fractions:
  - $a\left(\frac{b}{c}\right) = \frac{ab}{c}$
  - $\frac{\frac{a}{c}}{b} = \frac{a}{bc}$
  - $\frac{\frac{a}{c}}{\frac{b}{d}} = \frac{ad}{bc}$
  - $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
  - $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$

Axioms of Equality

- $a + b = c \implies a = c - b$
- $a - b = c \implies a = c + b$
- $ab = c \implies a = \frac{c}{b}$
- $\frac{a}{b} = c \implies a = bc$

Calculus

Introductory Example

- Function: how y is gotten from x, written as $y = f(x)$.
  - E.g., $y = 3x + 2$: if $x = 3$, then 3 times 3 and plus 2 will get $y = 11$.
- Differentiation: how the value of y changes when the value of x changes.
  - E.g., $y = 3x + 2$,

Table 1: Table for how the value of x affects the value of y

x	1	2	3	4	5
y	5	8	11	14	17

Notice $\Delta x = 1 \implies \Delta y = 3 \implies \frac{\Delta y}{\Delta x} = 3$, change to differentiation notation, $\frac{dy}{dx} = 3$

- **Tips**: $y = 3x^2 + 9x + 2$, look at terms with x, $dy = 3 \times 2x\,(dx) + 9\,(dx) \implies \frac{dy}{dx} = 6x + 9$

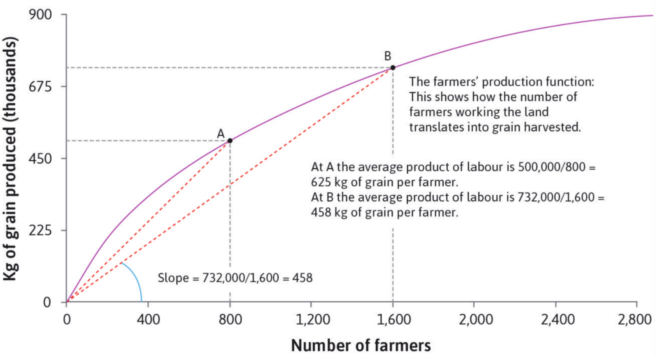
Notation and Convention

- Function is a mapping from argument to outcome:
  - $y = f(x)$: f describes a mapping from argument x to outcome y
- Differentiation: given mapping f, how much y would change (dy) if x change a fixed amoung (dx)
- First derivative: $y = f(x) \implies \frac{dy}{dx}$ or $f'(x)$
  - the "change" itself
  - **Example:** $y = x^{\alpha} \implies \frac{dy}{dx} = \alpha x^{\alpha-1}$
- Partial derivative: $y = f(x, z) \implies \frac{\partial y}{\partial x}$
  - **Example:** $y = x^{\alpha} z^{1-\alpha} \implies \frac{\partial y}{\partial x} = \alpha x^{\alpha-1} z^{1-\alpha}; \frac{\partial y}{\partial z} = (1-\alpha) x^{\alpha} z^{-\alpha}$
- Second derivative: $y = f(x) \implies \frac{d^2 f}{dx^2}$ or $f''(x)$
  - the speed of "change"
  - **Example:** $y = x^{\alpha} \implies \frac{d^2 f}{dx^2} = \alpha(\alpha-1) x^{\alpha-2}$

Production

Average Production of Labor (APL):

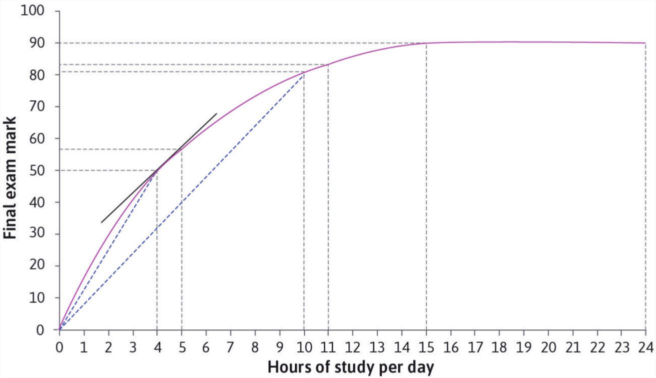
$$y = f(x) \implies APL = \frac{y}{x} = \frac{f(x)}{x}$$



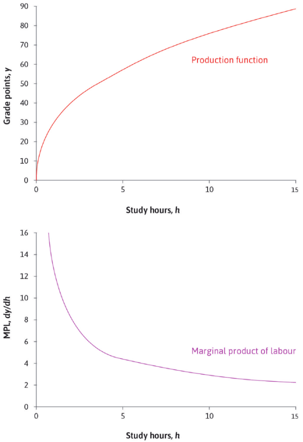
Production (Cont.)

Marginal Production of Labor (MPL):

$$y = f(x) \implies MPL = \frac{dy}{dx} = \frac{df(x)}{dx}$$



Concave / Convex and Diminishing MPL



- Concave v.s. Convex: Is production function looks like a "cave"?
- Concave function: whenever study hour increases by 1 unit, the speed of increase in grade point is decreasing.
  - $\Longrightarrow$ decreasing MPL

Application of Differentiation: Elasticity

Definition (The "A" Elasticity of "B")

percentage change in "B" when "A" changes by 1%, i.e., $-\frac{\%\Delta B}{\%\Delta A}$

Definition (The price elasticity of quantity demanded)

percentage change in quantity demanded when price changes by 1%, i.e., $-\frac{\%\Delta Q}{\%\Delta P}$

- Calculate percentage: $\frac{\text{value}}{\text{total amount}} \times 100\%$
- Expand the $\%\Delta$ part: $\%\Delta Q = \frac{\Delta Q}{Q}$
- Use differentiation notation: $\%\Delta Q = \frac{\Delta Q}{Q} = \frac{dQ}{Q}$
- Rewrite Def of elasticity: $-\frac{\%\Delta Q}{\%\Delta P} = -\frac{dQ}{Q}\Big/\frac{dP}{P} = -\frac{P}{Q}\frac{dQ}{dP}$

Technological Progress Example: Numerically calculating π

–The Discovery That Transformed Pi by Veritasium