Lecture 11 Distorting Taxes and the Welfare Theorems

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Overview

In previous lectures, all the taxes we are discussing is lump-sum tax.

- pure income effect, no change to consumption-leisure allocation
- satisfy both welfare theorems

In this lecture, the distorting taxes will include substitution effect, and thus

- creating "wedges" to distort consumption-leisure choice
- violate the welfare theorems (CE \neq SPP)

SPP in Simplified Model



Simplified Model

Full Model

Assume production is labor-only technology:

$$Y = zN^d$$

So PPF is

$$C = z(h-l) - G$$

Thus, SPP is

$$\max_{l} U(z(h-l) - G, l)$$

$$\text{OC:} \quad \frac{D_{l}U(C, l)}{D_{C}U(C, l)} = MRS_{l,C}$$

$$= MRT_{l,C} = z = MPN$$

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Labor Demand in Simplified Model

Figure 5.15 The Labor Demand Curve in the Simplified Model



$$\max_{N^d} zN^d - wN^d$$

FOC would be z = w (horizontal line)

- if z < w: negative profit for every worker hired, choose N^d = 0
- if z > w: positive profit for every worker hired, choose $N^d = \infty$
- only z = w possible, ∴ linear PPF in previous slide
 - "infinitely elastic" N^d

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Lecture 11

Competitive Equilibrium w/ Distorting Tax

A competitive equilibrium, with $\{z, G, K\}$ exogenous, is a list of endogenous prices and quantities $\{C, l, N^s, N^d, Y, \pi, w, t\}$ such that:

1 taking $\{w, \pi\}$ as given, the consumer solves

 $\max_{C,l,N^s} U(C,l) \quad \text{subject to} \quad C = w(1-t)N^s + \pi \quad \text{and} \quad N^s + l = h$

Simplified Model

2 taking w as given, the firm solves:

 $\max_{N^d,Y,\pi} \pi \quad \text{subject to} \quad \pi = Y - w N^d \quad \text{and} \quad Y = z N^d$

3 the government spends $G = wtN^s$

4 the labor market clears at the equilibrium wage, i.e. $N^s = N^d$

Effect of Distorting Tax

Since the tax is imposed on consumers/workers, it distorted the consumption-leisure decision:

$$MRS_{l,C} = w(1-t)$$

So in the equilibrium, it deviates from SPP:

$$MRS_{l,C} = w(1-t) < w = z = MPN = MRT_{l,C}$$

Result: CE and SPP lead to different allocation!

Simplified Model

Graphical Representation

Figure 5.16 Competitive Equilibrium in the Simplified Model with a Proportional Tax on Labor Income



SPP solution lies at point E:

- \overline{AB} : PPF, slope -z
- can reach indifference curve I₁
 CE solution lies at point H:

• \overline{DF} : consumer's budget line

- \blacksquare can only reach I_2
- ${\color{black}\bullet} \ {\color{black}\mathsf{proportional}} \ {\color{black}\mathsf{tax}} \Rightarrow N^s \downarrow$
- $N^{s} \downarrow \Rightarrow Y \downarrow$, but still need to meet *G*, so $C \downarrow$: gov't budget critical!

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Simplified Model

How Much Tax Revenue can be Generated?

equilibrium wage: w = z, implies total tax revenue by solve consumer problem:

Full Model





 $R(t) = tz(h - l^*(t)),$

What t maximizes? Solve

$$\max_{t} R(t) = \max_{t} tz(h - l^*(t)),$$

- not just t = 1! tax rate vs tax base
- t = 0: no revenue because no tax
- *t* = 1: no revenue because no incentive to work

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Full Model Elaboration

Let $U(C, l) = \ln C + \ln l$, and h = z = 1, by firm's problem we know w = z = 1. Consumer has some non-labor income denoted as x > 0. FOC leads to

$$MRS_{l,C} = \frac{C}{l}$$

= $\frac{(1-t)(1-l) + \pi}{l} = 1 - t < 1 = MRT_{l,C}$
 $\Rightarrow (1-t)(1-l) + \pi = (1-t)l$
 $\Rightarrow 1 - l + \frac{\pi}{1-t} = l \Rightarrow 2l = 1 + \frac{\pi}{(1-t)}$
 $\Rightarrow l = \frac{1}{2} + \frac{\pi}{2(1-t)}$
 $\Rightarrow N^{s}(t) = 1 - l = \frac{1}{2} - \frac{\pi}{2(1-t)}$

Simplified Model

Full Model

Maximize Tax Revenue

Total tax revenue is

$$R(t) = tN^s(t),$$

and thus government's problem is

$$\max_t \frac{1}{2}t - \frac{t\pi}{2(1-t)}.$$

FOC leads to

$$\frac{1}{2} - \frac{\pi(1-t) + t\pi}{2(1-t)^2} = 0 \Rightarrow \frac{1}{2} - \frac{\pi}{2(1-t)^2} = 0$$
$$\frac{1}{2} = \frac{\pi}{2(1-t)^2} \Rightarrow 1 = \frac{\pi}{(1-t)^2}$$
$$t = 1 - \sqrt{\pi}$$

Full Model

Visualization



Consider two cases:

() consumer is poor (low π)

② consumer is rich (high π) For a given after tax-wage , rich consumer supplies less labor

- tax revenue shifts down
- Laffer peak shifts left
- many other conditions also impact this analysis!

Multiple Competitive Equilibria Possible

Figure 5.18 Two Competitive Equilibria



Previous slide logic implies the government can choose 2 tax rates for a given required level of G

- both *t*₁ and *t*₂ yield the same revenue
- consumer strictly better off under lower tax rate t₁

Conclusion

We've focused on the simple case to keep analysis straightforward, but logic applies more broadly.

• SPP: $MRS_{l,C} = MRT_{l,C} = MPN$, since PPF is C = zF(K, N) - G

• CE: same distortion as our simple case:

- consumer problem implies $MRS_{l,C} = w(1-t)$
- firm problem implies $MRT_{l,C} = w$
- same result as simplified model: $MRS_{l,C} \neq MRT_{l,C}$, unlike SPP
- only difference from simplified model: $MPN = D_N F(K, N) \neq z$