Lecture 3:
Business Cycle Measurement

Hui-Jun Chen

National Tsing Hua University

September 10, 2025

Outline

Review: Trend and Cycle in Per Capital Real GDP

Figure: Figure 1.3: Natural log of Per Capita Real GDP and trend, 1900 − 2014: $y = \ln(Y)$, trend = HPFilter(y)
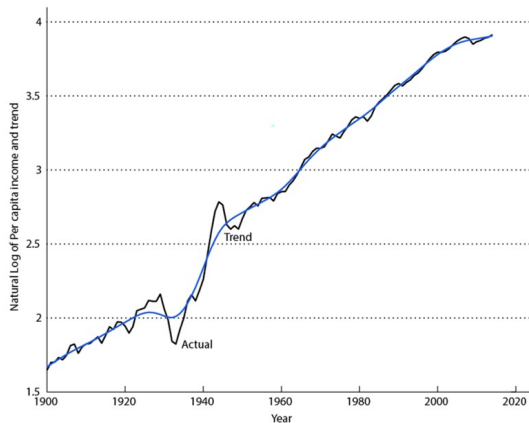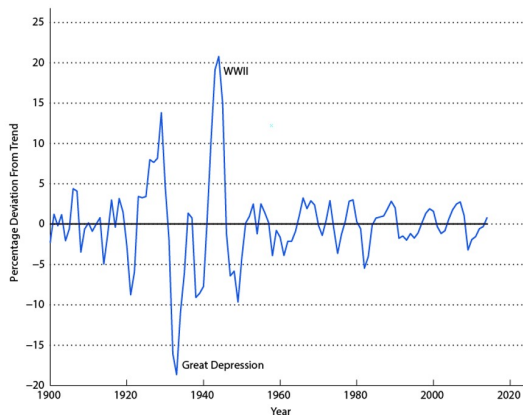
Figure: Figure 1.4 Percentage Deviation from Trend in Per Capita Real GDP: actual - trend

Statistical Concept to measure fluctuation

> **mean/average** $\bar{X}$: average level of variable X

$$\bar{X} = E(X) \approx \frac{1}{N} \sum_{i=1}^{N} X_i, \tag{1}$$

e.g. $X = \{1, 2, 3, 4, 5\}, \bar{X} = \frac{1+2+3+4+5}{5} = 3$

> **variance** σ_X^2: dispersion of variable X relative to mean

$$\sigma_X^2 = V(X) = E[(X - \bar{X})^2] \approx \frac{1}{N} \sum_{i=1}^{N} [(X_i - \bar{X})^2], \tag{2}$$

e.g. $\sigma_X^2 = \frac{(1-3)^2+(2-3)^2+(3-3)^2+(4-3)^2+(5-3)^2}{5} = \frac{4+1+0+1+4}{5} = 2$
standard deviation is $\sigma_X = \sqrt{\sigma_X^2}$.

Statistical Concept to measure fluctuation (Cont.)

> **covariance** σ_{XY}: dispersion of variable X "associates" with Y.

$$\sigma_{XY} = E[(X - \bar{X})(Y - \bar{Y})] \approx \frac{1}{N} \sum_{i=1}^{N} [(X_i - \bar{X})(Y_i - \bar{Y})], \tag{3}$$

e.g. $Y = \{2, 3, 4, 5, 6\}$, $\bar{Y} = 4$,
$\sigma_{XY} = \frac{(1-3)(2-4)+(2-3)(3-4)+(3-3)(4-4)+(4-3)(5-4)+(5-3)(6-4)}{5} = 2$

> **correlation** ρ_{XY}: level of association between variables X and Y

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}, \tag{4}$$

e.g. $\rho_{XY} = \frac{2}{\sqrt{2} \times \sqrt{2}} = 1$
$\rho_{XY} > 0$: X and Y move together; < 0: move opposite directions

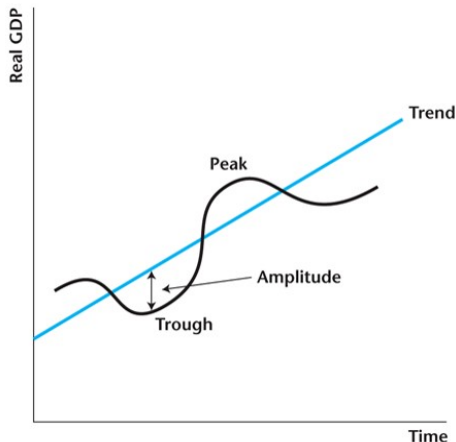# Outline

GDP Fluctuations: Definition and Measurement

- **Business cycle**: fluctuations about trend in real GDP
- **Booms/Expansions**: persistent positive deviation
- **Recessions**: persistent negative deviation
- **Peak/Trough**: turning points on deviations
- **Amplitude**: size of deviations
- **Frequency**: # of peaks per year

Figure: Figure 3.1 Business Cycle

Measurement on Comovement

Three key concepts to measure comovement of variables X with GDP Y:

1. **relative volatility** σ_X/σ_Y: how noisy is X relative to GDP
   - ❯❯ σ_X/σ_Y high $\Rightarrow X$ is noisy

2. **cyclicality** ρ_{XY}: how does X comove with GDP?
   - 2.1 procyclical: $\rho_{XY} > 0$, X and GDP comove together
   - 2.2 acyclical: $\rho_{XY} \approx 0$, X and GDP not comoving
   - 2.3 countercyclical: $\rho_{XY} < 0$, X and GDP move in opposite direction

3. **lead/coincident/lag**: does X predict GDP (lead), the opposite (lag) or nor (coincident)?
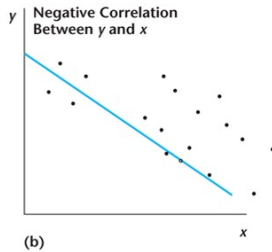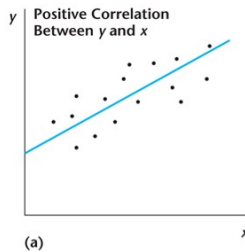
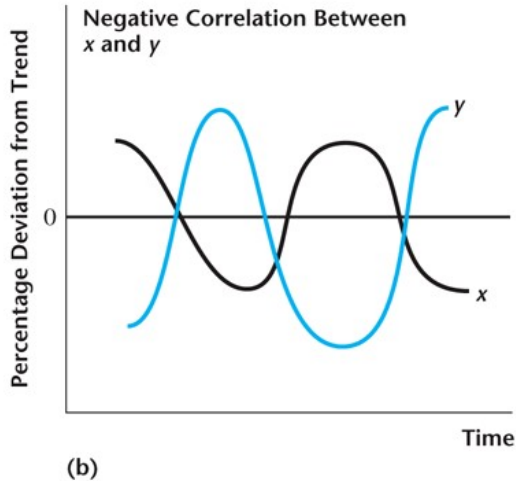# Outline

Visualization of Correlation: Scatter Plots
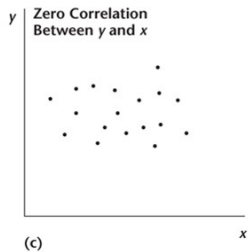
Visualization of Correlation: Time Series

(a) Positive Correlation Between x and y

(b) Negative Correlation Between x and y
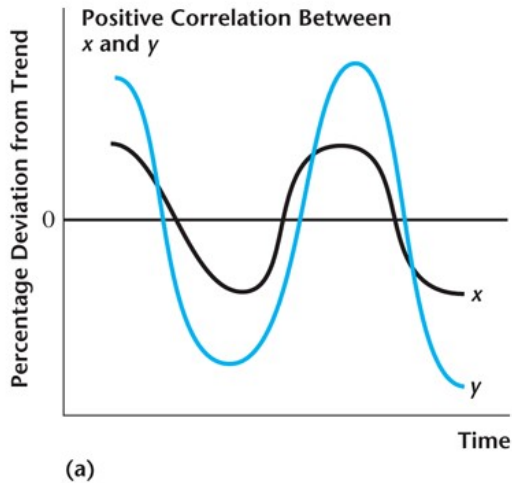
Visualization of Leading and Lagging: Time Series
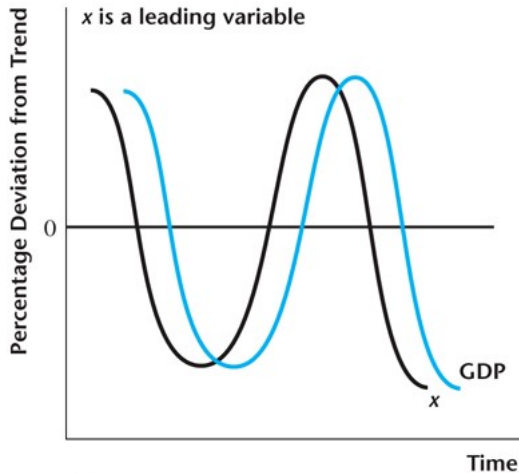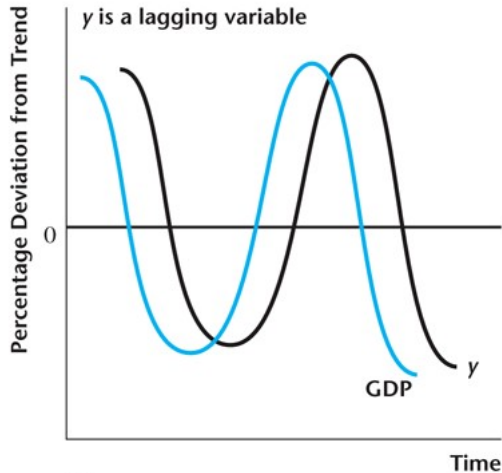
Outline

Data: Comovement of C and I

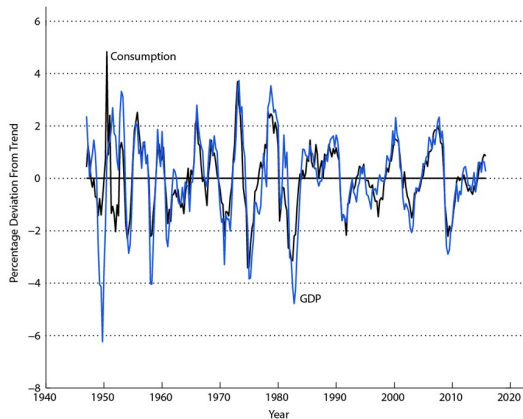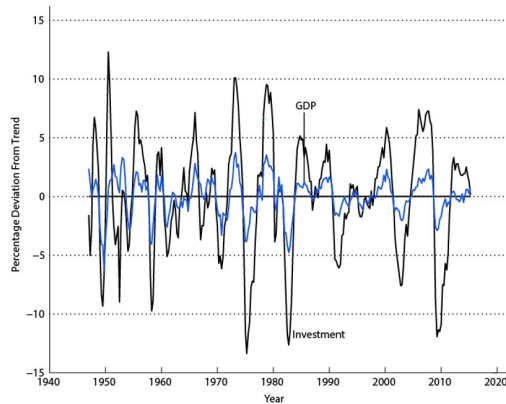Figure: Figure 3.9 Percentage Deviations from Trend in Real Consumption and Real GDP

Figure: Figure 3.10 Percentage Deviations from Trend in Real Investment and Real GDP

Data: Correlation of Imports
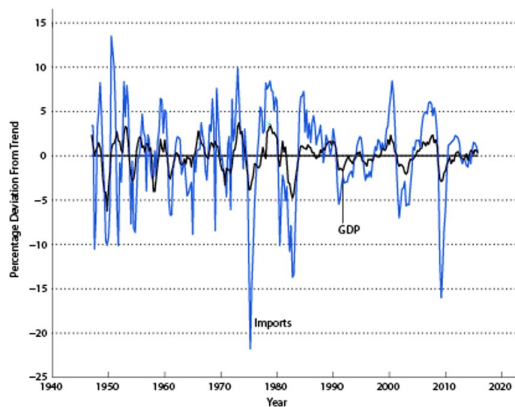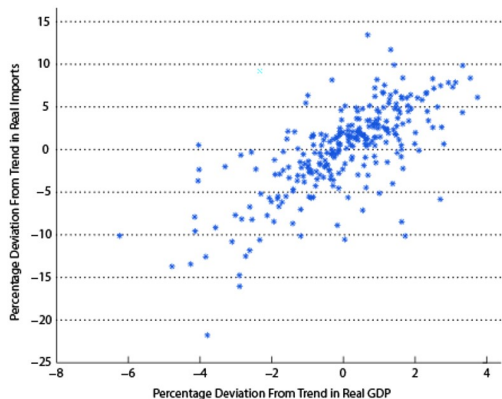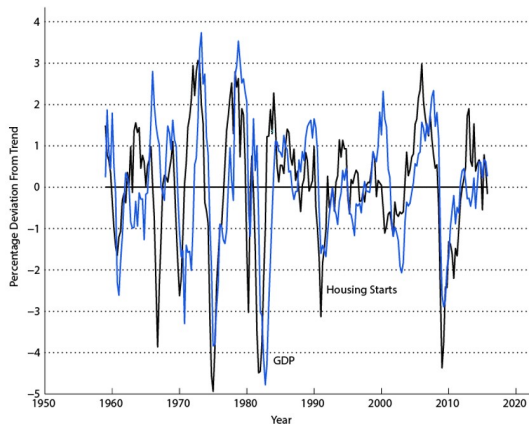
Figure: Figure 3.5 Time Series (Detrended)

Figure: Figure 3.6 Scatter Plot: Time Series

Data: Leading Indicator, Housing Starts

Figure: Figure 3.8 Percentage Deviations in Real GDP and Housing Starts

- **Def**: the construction project is started for a private dwelling
- Question: Why housing start is predicting GDP?
- Ans: commitment to a quantity of residential investment

Summary

Table: Business cycle statistics relative to GDP

Variable	Corr. Coef.	Std Dev (% of S.D. GDP)	Cyclicality	Lead/Lag	Variation Relative to GDP
consumption	0.77	77	Procyclical	Coincident	Smaller
investment	0.80	301	Procyclical	Coincident	Larger
employment	0.78	65	Procyclical	Lagging	Smaller
average labor productivity	0.77	63	Procyclical	Coincident	Smaller