Lecture 4
Representative Consumer
Preference and Constraints

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Overview: Lecture 4 - 7

Provide micro-foundation for the macro implication (Lucas critique)

- **Representative Consumer:**
  - Lecture 4: preference, constraints
  - Lecture 5: optimization, application
  - Lecture 6: Numerical Examples
- **Representative Firm:**
  - Lecture 7: production, optimization, application

Outline

Utility Function

We use utility function $U(C, l)$ to represent the preference/happiness

- C: consumption (assume single/composite goods)

- l: leisure (time spent not working)

Utility function defines the ranking of (C, l) bundles

- If $U(C_1, l_1) > U(C_2, l_2)$, then (C_1, l_1) is strictly preferred to (C_2, l_2)
  - $\because (C_1, l_1)$ bundle generate more happiness than (C_2, l_2) bundle

- If $U(C_1, l_1) = U(C_2, l_2)$, then indifferent between (C_1, l_1) and (C_2, l_2)
  - $\because (C_1, l_1)$ bundle generate same happiness as (C_2, l_2) bundle

- Note: level of utility is meaningless, only order matters!

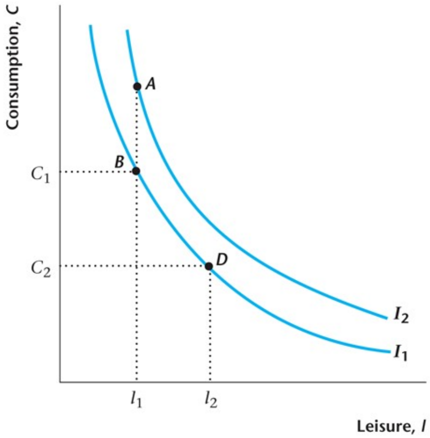
Properties of Utility Function

1. **Monotonicity**: more is always better!
   - If $C_1 > C_2$ and $l_1 > l_2$, then $U(C_1, l_1) > U(C_2, l_2)$
2. **Convexity**: prefer diversified consumption bundles
   - e.g. prefer food + leisure rather than overeating / oversleeping
3. **Normality**: consumption and leisure are normal goods
   - income $\uparrow \Rightarrow$ consumption $\uparrow$
   - leisure is complicated: relates to income
     - the poor: less leisure means more labor income
     - the rich: more income means more leisure

Rep. of Utility Function: Indifference Curve

- **Def:** (C, l) bundles that yield the same utility level
- Monotonicity $\Rightarrow$ downward sloping
- Convexity $\Rightarrow$ diversity shown in comparison between point B and D

Figure: Figure 4.1 Indifference Curves



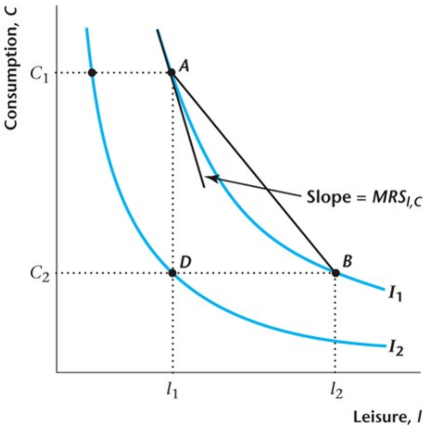
Rep. of Utility Function: Indifference Curve (Cont.)

- Normality: Marginal Rate of Substitution
  - **Marginal**: for arbitrary small change in x-axis (leisure in this case)
  - **rate of substitution**: the amount on y-axis has to be sacrificed (consumption in this case)

$$MRS_{l,C} = \frac{D_l U(C,l)}{D_C U(C,l)}, \quad (1)$$

where $D_x U(\cdot)$ is derivative of U w.r.t. x

Figure: Figure 4.2 MRS

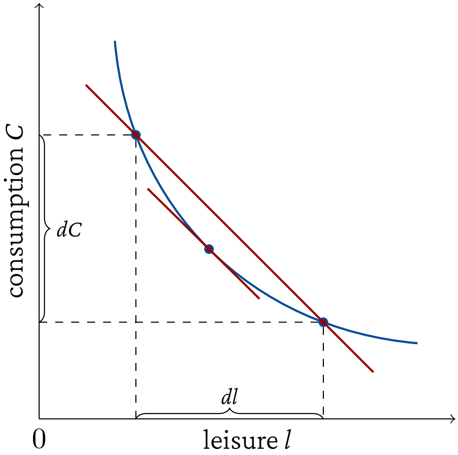


Computing MRS

- little change in leisure $dl > 0 \Rightarrow$ change in utility $D_l U(C, l) dl$
- with the cost of income loss $\Rightarrow$ consumption has to drop by $dC < 0$ amount $\Rightarrow$ change in utility $D_C U(C, l) dC$
- Stay on the IC $\Rightarrow$ utility remain the same:

$$D_C U(C, l) dC + D_l U(C, l) dl = 0$$

$$\frac{dC}{dl} = -\frac{D_l U(C, l)}{D_C U(C, l)} = -MRS_{l,C}$$



Algebraic Example

Suppose $U(C,l) = \frac{C^{1-\sigma}}{1-\sigma} + \psi \ln l$, where σ and ψ are parameters. Then,

- $D_C U(C,l) = (1-\sigma)\frac{C^{1-\sigma-1}}{1-\sigma} = C^{-\sigma}$
- Remember $\frac{d \ln l}{dl} = \frac{1}{l}$, $D_l U(C,l) = \frac{\psi}{l}$
- $MRS_{l,C} = \frac{D_l U(C,l)}{D_C U(C,l)} = \frac{\psi}{l C^{-\sigma}}$

Outline

Budget Constraints

- **Time**: consumer has h hours per day, and allocate between leisure l and labor supply N^s

$$l + N^s = h \tag{2}$$

- **Budget**: consumer cannot spend more than the income he/she has
  - labor income: wage rate w times labor supply N^s, wN^s
  - dividends income: consumer buys share of the firm, gain dividend π
  - tax: consumer is subject to lump-sum taxes T

$$C \leq wN^s + \pi - T \tag{3}$$

- Consumption is **numeraire**: price normalized to 1.
  - Imagine consumption goods as unit of account, ppl directly trade with consumption goods

Visualization of Budget Set

Figure: Figure 4.3 Representative Consumer's Budget Constraint when $T > \pi$ ("poor")

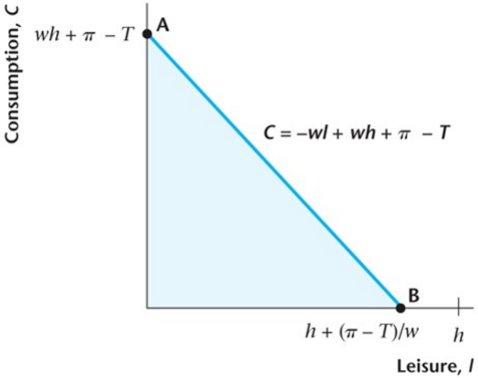
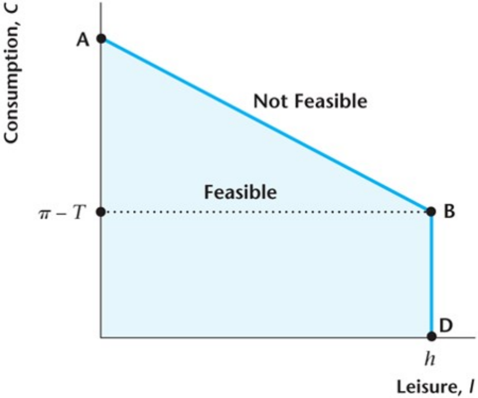


Figure: Figure 4.4 Representative Consumer's Budget Constraint when $T < \pi$ ("rich")



Outline

Note on Calculus

- Function: $y = f(x)$, how y is determined by x
  - E.g., $y = 3x + 2$: if $x = 3$, then 3 times 3 and plus 2 will get $y = 11$

- Differentiation: how changes in x results in change in y
  - E.g., $y = 3x + 2$,

Table: Table for how the value of x affects the value of y

x	1	2	3	4	5
y	5	8	11	14	17

Notice $\Delta x = 1 \implies \Delta y = 3 \implies \frac{\Delta y}{\Delta x} = 3$, change to differentiation notation, $\frac{dy}{dx} = 3$

- **Tips**: $y = 3x^2 + 9x + 2$, look at terms with x, $dy = 3 \times 2x\,(dx) + 9\,(dx) \implies \frac{dy}{dx} = 6x + 9$