

Lecture 9

Social Planner's Problem

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Overview

After constructing both **consumers'** and **firms'** problem, we start to bring them together in **one-period model**:

- Lecture 8: **competitive equilibrium** (CE)
 - each agent solve their problems individually
 - aggregate decision determines “prices” (wage, rent, etc.)
- Lecture 9: **social planner's problem** (SPP)
 - imaginary and benevolent social planner determines the allocation
 - should be the most efficient outcome
- Lecture 10: CE and SPP examples

What is Social Planner?

- **Benevolent dictator** whose goal is to maximize **social welfare** given **technological constraint**
- **Social welfare**: joint “happiness” of every agent in this economy
 - » **consumer**: tangency between IC and budget line in (C, l) -plane
 - » **firm**: $Y = zF(K, N) = zF(K, h - l)$
 - labor market clearing: $N = N^s = N^d$
 - consistent with consumer behavior: $N = h - l$
 - » **government**: income-expenditure identity, $C = Y - G$
 - government is not necessary the social planner! (also one of the agents)
- **Technological constraint**: production possibility frontier

Outline

1 PPF

2 Pareto Efficiency

3 Social Planner

Production Possibility Frontier (PPF)

- › Def: technological possibilities for the whole economy

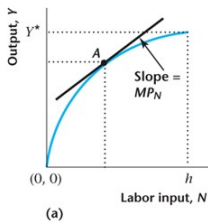
$$C = zF(K, h - l) - G \quad (1)$$

- › **Marginal rate of transformation (MRT)**: rate to transform leisure to consumption (through work)

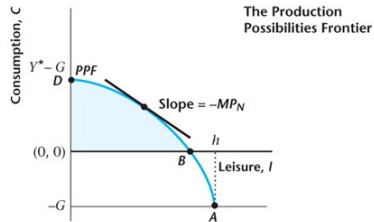
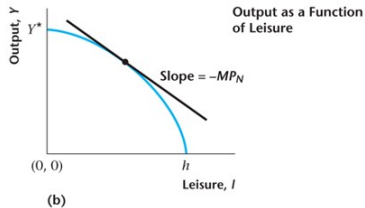
$$\begin{aligned} MRT_{l,C} &= zD_N F(K, N) \\ &= MPN \end{aligned} \quad (2)$$

Production Possibility Frontier (Figure)

Figure: The Production Function and the Production Possibilities Frontier

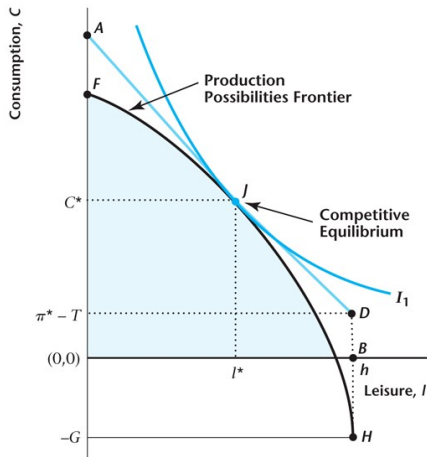


Production Function



Competitive Equilibrium: Graphical Representation

Figure: Competitive Equilibrium



Combine PPF with IC:

- \overline{AD} : tangent to consumer's IC I_1 and PPF \overline{FH}
- negative slope of \overline{AD} : equilibrium wage w
 - » $\therefore \overline{AD}$ is budget line
- Recall Lecture 8 & last slide:
 - » consumer: $MRS_{l,C} = w$
 - » firm: $MPN = w$
 - » efficiency: $MRT_{l,C} = MPN$

$$MRS_{l,C} = MRT_{l,C} = MPN$$

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1 PPF

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Concept: Pareto Improvement / Optimal

A competitive equilibrium is **Pareto optimal** or **Pareto efficient** if there is no way to **rearrange production or to reallocate goods** so that **someone is made better off without making someone else worse off**.

- only one consumer, so relatively straightforward
- but, still a powerful concept:
 - free markets can produce socially efficient outcomes
 - often easier to analyze social optimum than competitive equilibrium
- caveats:
 - “efficiency” in economics is a statement about a model
 - very narrow: e.g. having Jeff Bezos pay for a meal for someone in need is “harming” Bezos.

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Social Planner's Problem

objective: consumer's utility $\max_{C,l,N,Y} U(C, l)$

subject to

agg. resource constraint $C + G \leq Y$

production constraint $Y = zF(K, N)$

labor constraint $N = h - l$

- **What's here:** GDP accounting, physical / technological constraints, required government spending, consumer preferences
- **What's not:** consumer's budget constraint, the wage rate, consumer's / firm's individual problems, profits, taxes

Solving Social Planner's Problem

We know all constraints bind, so by substituting:

$$\max_l U(zF(K, h - l) - G, l) \quad (3)$$

FOC:

$$\begin{aligned} D_l U(zF(K, h - l) - G, l) \\ = D_C U(zF(K, h - l) - G, l) (zD_N F(K, h - l)) \end{aligned} \quad (4)$$

Rearrange:

$$\frac{D_l U(zF(K, h - l) - G, l)}{D_C U(zF(K, h - l) - G, l)} = zD_N F(K, h - l) \Rightarrow MRS_{l,C} = MRT_{l,C} \quad (5)$$

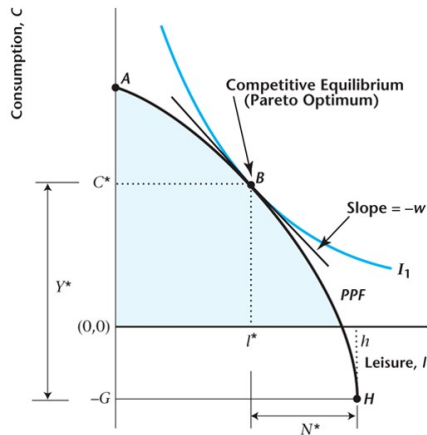
Same Result! Why?

Welfare Theorem

- **First welfare theorem:** under certain conditions, the allocation under a competitive equilibrium is Pareto optimal
- **Second welfare theorem:** under certain conditions, a Pareto optimal allocation is the allocation for a competitive equilibrium.
- straightforward to show here (we already have!), but not always so as conditions not always met!
- SPP and CE often alike if not identical, serves as a good benchmark

Social Planner's Problem: Graphical Representation

Figure: Pareto Optimality



Apply SPP & 2nd welfare theorm for competitive equilibrium:

- l^* determined by SPP at B
- C^*, N^*, Y^* by plugging into constraints
- $w^* = MPN = MRT_{l,C} = MRS_{l,C}$

What Can Go Wrong? Cases when $SPP \neq CE$

1. Externalities: activity for which an individual does not take account of all associated costs and benefits: can be positive or negative
 - » example: pollution must be cleaned up, but firm doesn't have to
2. Distorting taxes: lead to “wedges” between MRS, MP, and MRT
 - » example: proportional labor income tax vs lump-sum tax
3. Non-competitive / monopolistic behavior: firms or consumers may not be price takers
 - » examples: local media markets, negotiations