Lecture 11
Distorting Taxes and the Welfare Theorems

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October 16, 2025

Overview

In previous lectures, all the taxes we are discussing is lump-sum tax.

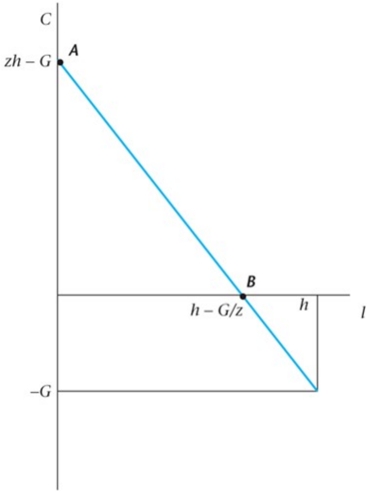
- pure income effect, no change to consumption-leisure allocation
- satisfy both welfare theorems

In this lecture, the distorting taxes will include substitution effect, and thus

- creating “wedges” to distort consumption-leisure choice
- violate the welfare theorems (CE $\neq$ SPP)

Outline

SPP in Simplified (but Problematic) Model



Assume production is labor-only technology:

$$Y = zN^d$$

So PPF is

$$C = z(h - l) - G$$

Thus, SPP is

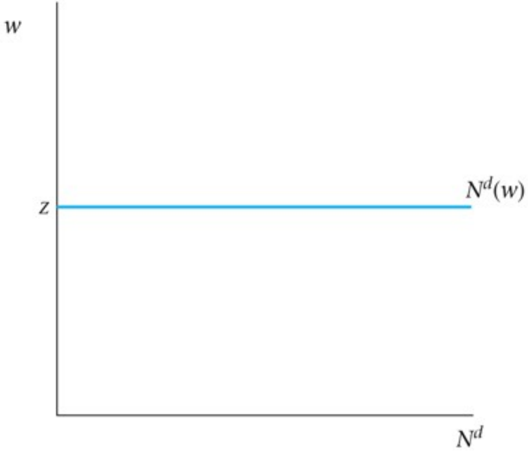
$$\max_{l} U(z(h-l) - G, l)$$

$$\text{FOC:} \quad \frac{D_l U(C,l)}{D_C U(C,l)} = MRS_{l,C}$$

$$= MRT_{l,C} = z = MPN$$

Labor Demand in Simplified Model

Figure: Figure 5.15 The Labor Demand Curve in the Simplified Model



$$\max_{N^d} zN^d - wN^d$$

FOC would be $z = w$ (horizontal line)

- if $z < w$: negative profit for every worker hired, choose $N^d = 0$
- if $z > w$: positive profit for every worker hired, choose $N^d = \infty$
- only $z = w$ possible, $\therefore$ linear PPF in previous slide
  - "infinitely elastic" N^d

Competitive Equilibrium w/ Distorting Tax

A competitive equilibrium, with $\{z, G\}$ exogenous, is a list of endogenous prices and quantities $\{C, l, N^s, N^d, Y, \pi, w, t\}$ such that:

1. taking $\{w, \pi\}$ as given, the consumer solves

$$\max_{C,l,N^s} U(C, l) \quad \text{subject to} \quad C = w(1-t)N^s + \pi \quad \text{and} \quad N^s + l = h$$

2. taking w as given, the firm solves:

$$\max_{N^d,Y,\pi} \pi \quad \text{subject to} \quad \pi = Y - wN^d \quad \text{and} \quad Y = zN^d$$

3. the government spends $G = wtN^s$

4. the labor market clears at the equilibrium wage, i.e. $N^s = N^d$

Effect of Distorting Tax

Since the tax is imposed on consumers/workers, it distorted the consumption-leisure decision:

$$MRS_{l,C} = w(1-t)$$

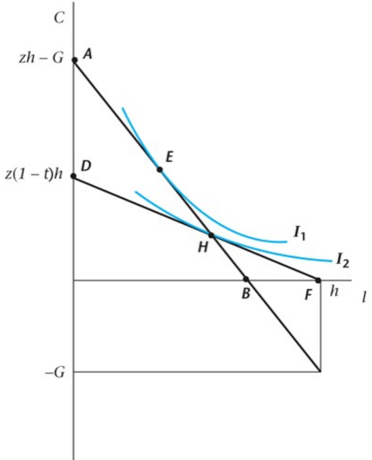
So in the equilibrium, it deviates from SPP:

$$MRS_{l,C} = w(1-t) < w = z = MPN = MRT_{l,C}$$

Result: CE and SPP lead to different allocation!

Graphical Representation

Figure: Figure 5.16 Competitive Equilibrium in the Simplified Model with a Proportional Tax on Labor Income



SPP solution lies at point E:

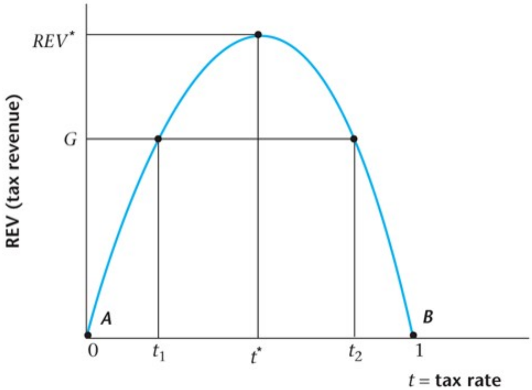
- $\overline{AB}$: PPF, slope $-z$
- can reach indifference curve I_1

CE solution lies at point H:

- $\overline{DF}$: consumer’ s budget line
- can only reach I_2
- proportional tax $\Rightarrow N^s \downarrow$
- $N^s \downarrow \Rightarrow Y \downarrow$, but still need to meet G, so $C \downarrow$: gov’ t budget critical!

How Much Tax Revenue can be Generated?

Figure: Figure 5.17 A Laffer Curve



equilibrium wage: $w = z$, implies total tax revenue by solve consumer problem:

$$R(t) = tz(h - l^*(t)),$$

What t maximizes? Solve

$$\max_t R(t) = \max_t tz(h - l^*(t)),$$

- not just $t = 1$! tax rate vs tax base
- $t = 0$: no revenue because no tax
- $t = 1$: no revenue because no incentive to work

Back

Full Model Elaboration

Let $U(C,l) = \ln C + \ln l$, and $h = z = 1$, by firm's problem we know $w = z = 1$. Consumer has some non-labor income denoted as $x > 0$. FOC leads to

$$\begin{aligned}
MRS_{l,C} &= \frac{C}{l} \\
&= \frac{(1-t)(1-l)+\pi}{l} = 1-t < 1 = MRT_{l,C} \\
&\Rightarrow (1-t)(1-l)+\pi = (1-t)l \\
&\Rightarrow 1-l+\frac{\pi}{1-t} = l \Rightarrow 2l = 1+\frac{\pi}{(1-t)} \\
&\Rightarrow l = \frac{1}{2}+\frac{\pi}{2(1-t)} \\
&\Rightarrow N^s(t) = 1-l = \frac{1}{2}-\frac{\pi}{2(1-t)}
\end{aligned}$$

Maximize Tax Revenue

Total tax revenue is

$$R(t) = tN^s(t),$$

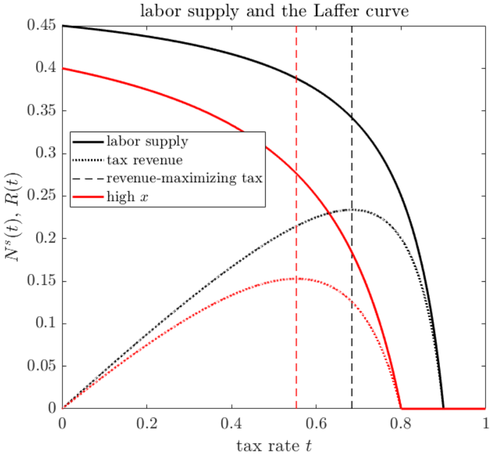
and thus government's problem is

$$\max_t \frac{1}{2}t - \frac{t\pi}{2(1-t)}.$$

FOC leads to

$$\begin{aligned}\frac{1}{2} - \frac{\pi(1-t)+t\pi}{2(1-t)^2} = 0 \Rightarrow \frac{1}{2} - \frac{\pi}{2(1-t)^2} &= 0\\ \frac{1}{2} = \frac{\pi}{2(1-t)^2} \Rightarrow 1 &= \frac{\pi}{(1-t)^2}\\ t &= 1-\sqrt{\pi}\end{aligned}$$

Visualization



Consider two cases:

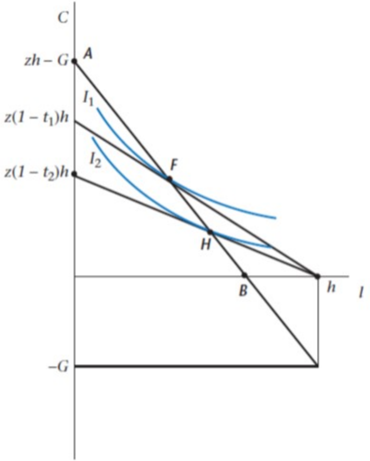
1. consumer is poor (low π)
2. consumer is rich (high π)

For a given after tax-wage , rich consumer supplies less labor

- tax revenue shifts down
- Laffer peak shifts left
- many other conditions also impact this analysis!

Multiple Competitive Equilibria Possible

Figure: Figure 5.18 Two Competitive Equilibria



Previous slide logic implies the government can choose 2 tax rates for a given required level of G

- both t_1 and t_2 yield the same revenue
- consumer strictly better off under lower tax rate t_1

What's wrong with this model?

Recall that $Y = zN^d$ implies labor supply $N^s(t)$ equals to

$$N^s(t) = 1 - l = \frac{1}{2} - \frac{\pi}{2(1-t)}, \tag{1}$$

and the total tax revenue is given by

$$R(t) = wtN^s(t). \tag{2}$$

In equilibrium $w = z = 1$, so $\pi = zN^d - wN^d = 0$, so this question is trivial...Stay tuned with Problem Sets 😉

Conclusion

We've focused on the simple case to keep analysis straightforward, but logic applies more broadly.

- SPP: $MRS_{l,C} = MRT_{l,C} = MPN$, since PPF is $C = zF(K, N) - G$
- CE: same distortion as our simple case:
  - consumer problem implies $MRS_{l,C} = w(1 - t)$
  - firm problem implies $MRT_{l,C} = w$
  - same result as simplified model: $MRS_{l,C} \neq MRT_{l,C}$, unlike SPP
  - only difference from simplified model: $MPN = D_N F(K, N) \neq z$