Lecture 12: Two-Period Consumer Problem

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Outline

Variables and Notation

Assume that consumer do NOT make consumption-leisure decision, but receive endowment of non-labor income y and subject to lump-sum tax t.

- y & t: today (date 0), and y' & t': tomorrow (date 1)
- in general, having a prime "$'$" represents tomorrow

If there's a saving technology exists (may not be available!), then consumer saves s today for tomorrow consumption, i.e.,

$$c + s \leq y - t,$$

where $s > 0$ represents "saver", and $s < 0$ represents "borrower".

Savings and the Credit Market

Buying/selling Bonds are the way to achieve saving s.

- lenders/savers buy bonds; borrowers sell bonds.

Consumer will get $1 + r$ unit of consumption goods tomorrow if he/she buys 1 unit of bond today, and thus tomorrow's budget constraint is

$$c' = y' - t' + (1 + r)s,$$

where r is the (net) real interest rate, and "$=$" since no date 2.

- relative price of consumption between today and tmw: $\frac{1}{1+r}$
- no default on bonds
- no middle man: bonds are trade directly between savers and borrowers

The Lifetime Budget Constraint

$$
\begin{aligned}
\text{Date 0}: &\quad c + s = y - t \\
\text{Date 1}: &\quad c' = y' - t' + (1+r)s \\
\text{Saving}: &\quad \Rightarrow s = \frac{c' - y' + t'}{1+r} \\
\text{Plug saving back to Date 0}: &\quad c + \frac{c' - y' + t'}{1+r} = y - t \\
\text{Rearrange}: &\quad \underbrace{c + \frac{c'}{1+r}}_{(1)} = \underbrace{y - t + \frac{y' - t'}{1+r}}_{(2)};
\end{aligned}
$$

- (1): present value of total lifetime consumption (choice by consumer)
- (2): present value of total lifetime net worth, also called *we* (fixed).

Numerical Example of Present Value

Suppose we have data:

y	y'	t	t'	r
110	120	20	10	0.1

The face value of the net worth is

$$y - t + y' - t' = 110 - 20 + 120 - 10 = 200$$

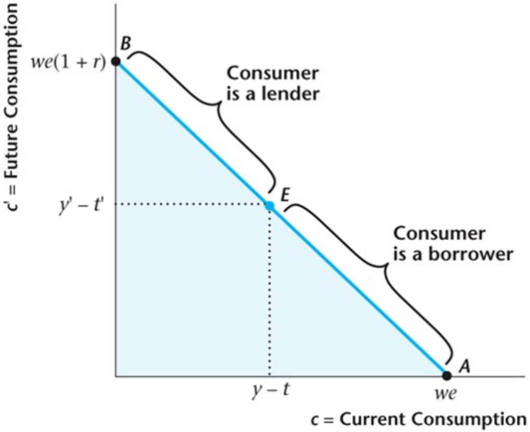
The present value of lifetime the net worth is

$$y - t + \frac{y' - t'}{1 + r} = 110 - 20 + \frac{120 - 10}{1.1} = 190$$

Future part has discounted 10% to be evaluated in consumption goods today.

Visualization: Lifetime Budget Constraint

Figure: Figure 9.1 Consumer' s Lifetime Budget Constraint



On (C, C') plane, $\because$ substitution between current and future consumption.

$$c' = \underbrace{we(1+r)}_{\text{y-intercept}} \underbrace{-(1+r)}_{\text{slope}} c$$

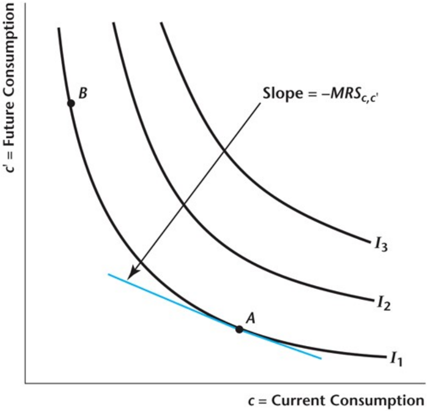
- E: endowment point, where $c = y - t$, and $c' = y' - t'$.
- $\overline{BE}$: lending, give up c for c'
- $\overline{AE}$: borrowing, the opposite

Outline

Consumer Preference in Two-Period Model

Since it is substitution between (c, c'), utility is $U(c, c')$, so

Figure: 9.2 A Consumer' s Indifference Curves



1. **monotonicity**: more is preferred
   - slope $= -MRS_{c,c'}$ (substitution)
   - $U(I_3) > U(I_2) > U(I_1)$
2. **convexity**: diversity is preferred
   - Is bow in towards the origin
   - consumption smoothing: preferred equal amount of (c, c')
3. **normality**: if lifetime wealth $\uparrow$, both c and $c' \uparrow$

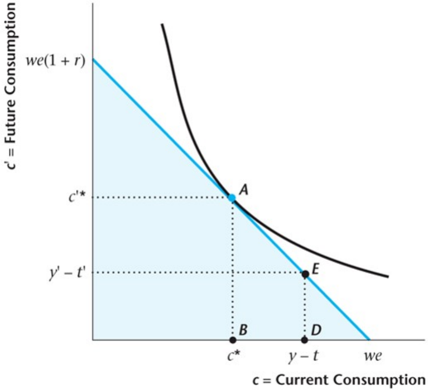
Outline

1. Budget
2. Preference
3. Consumer's Problem
4. Experiment
5. Data

Consumer's Problem: Two-Period Model

$$\max_{c,c'} U(c,c') \quad \text{subject to} \quad c' = we(1+r) - c(1+r)$$

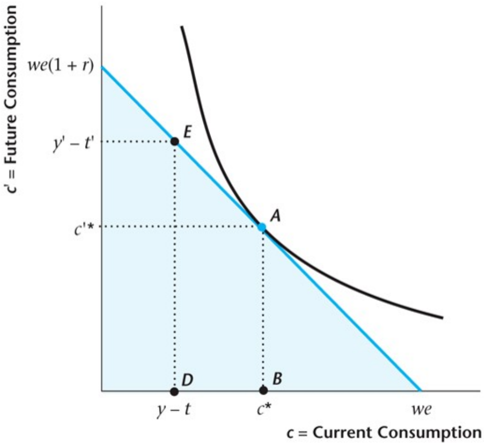
Figure: 9.3 A Consumer Who Is a Lender



- substitute c': $\max_{c} U(c, we(1+r) - c(1+r))$
- FOC: $D_c U(c,c') + D_{c'} U(c,c')(-(1+r)) = 0$
- rearrange: $\dfrac{D_c U(c,c')}{D_{c'} U(c,c')} = MRS_{c,c'} = 1 + r$
- Net worth at pt E: excess endowment at date 0, so saving $s = y - t - c^* > 0$!
- $c^* < y - t; c'^* > y' - t'$

Numerical Example

Figure: 9.3 A Consumer Who Is a Borrower



Let $U(c, c') = \ln c + \ln c'$ and $r = 0$,

$$MRS_{c,c'} = \frac{1/c}{1/c'} = \frac{c'}{c} = 1 + r = 1$$

optimal bundle: $c^* = c'^*$

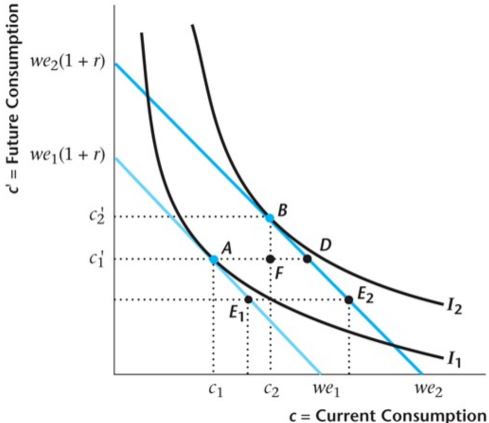
- if $we = 1 \Rightarrow c + c' = 1 \Rightarrow c^* = c'^* = \frac{1}{2}$
- if $E = (3/4, 1/4)$: consumer saves (last slide)
- if $E = (1/4, 3/4)$: consumer borrows

Outline

Increase in Current income

Let consumer's current income increases from y_1 to $y_2, y_2 > y_1$

Figure: 9.5 The Effects of an Increase in Current Income for a Lender

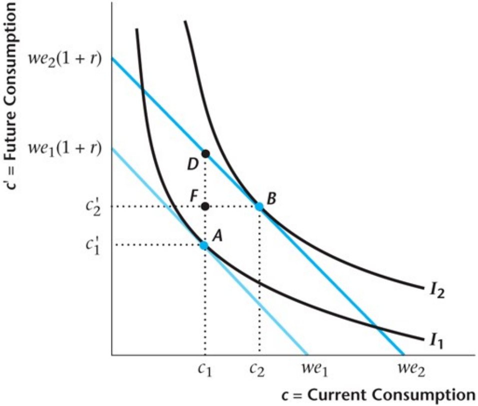


- parallel shift in budget line: r the same
- endowment: E_1 to E_2
- optimal bundle: A to B
- consumption smoothing: $c_1 = c_1', c_2 = c_2'$
- normality: $c_2 > c_1$, and $c_2' > c_1'$
- To support normality, $s_2 > s_1$

Increase in Future income

Let consumer's future income increases from y'_1 to $y'_2, y'_2 > y'_1$

Figure: 9.8 The Effects of an Increase in Future Income



- shift in lifetime wealth:
  $$\Delta we = we_2 - we_1 = \frac{y'_2 - y'_1}{1+r}$$
- optimal bundle: A to B
- consumption smoothing: $c_1 = c'_1, c_2 = c'_2$
- normality: $c_2 > c_1$, and $c'_2 > c'_1$
- To support normality, $s_2 < s_1$, shift income from date 1 to date 0!

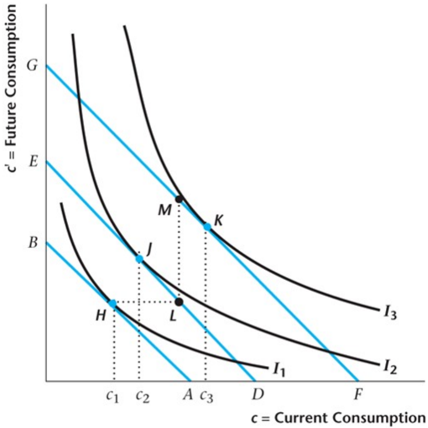
Intuition: Temporary vs Permanent Change in Income

Permanent Income Hypothesis (PIH): changes in income that are permanent have large effects on permanent income (lifetime wealth) and current consumption.

- temporary change in income: $y_1 \rightarrow y_2$ or $y_1' \rightarrow y_2'$
- permanent change in income: $y_1 \rightarrow y_2$ and $y_1' \rightarrow y_2'$
- intuition: permanent change compounds through lifetime
- most of temporary increase saved (e.g. COVID stimulus), yet more permanent increase is consumed (e.g. Rich ppl buys houses)

Visualization: Permanent Income Hypothesis

Figure: 9.9 Temporary Versus Permanent Increases in Income



Temporary:

- budget line: $\overline{AB} \rightarrow \overline{DE}$
- optimal bundle: $H \rightarrow J$

Permanent:

- budget line: $\overline{AB} \rightarrow \overline{GF}$
- optimal bundle: $H \rightarrow K$

In conclusion,

- larger effect on current consumption when change is permanent
- temporary $\Rightarrow$ saving; not necessary for permanent

Outline

Consumption Smoothing in Data

If all consumers act to smooth their consumption relative to their income, then aggregate consumption should likewise be smooth relative to aggregate income.

- recall relative volatility: expect $\sigma_C/\sigma_Y < 1$

There are three main components of aggregate consumption:

1. **non-durables**: e.g. food, dishes...
2. **durables**: e.g. cars, computers...
3. **services**: haircuts, repairing...

Does our prediction match the data in aggregate consumption? How about prediction with each component?

Durables Behaves Similar to Investment

Figure: 9.6 Percentage Deviations from Trend in Consumption of Durables and Real GDP, blue: Durables, black: GDP

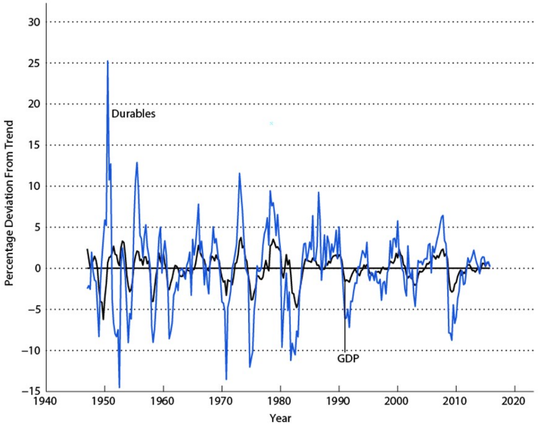
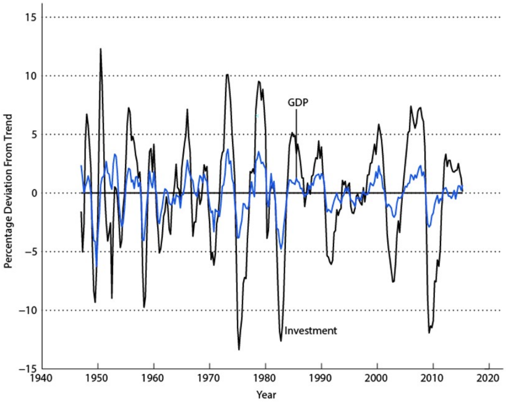


Figure: 3.10 Percentage Deviations from Trend in Real Investment and Real GDP, blue: GDP, black: investment



Non-Durables & Services Similar to Agg. Consumption

Figure: 9.7 Percentage Deviations from Trend in Consumption of Nondurables and Services and Real GDP, blue: GDP, lightblue: Nondurables + Service

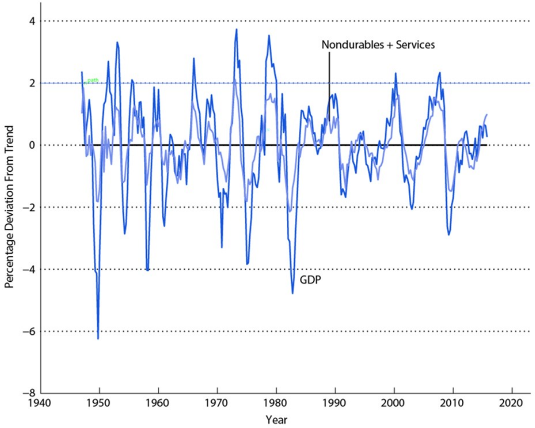


Figure: 3.9 Percentage Deviations from Trend in Real Consumption and Real GDP, blue: GDP, black: consumption