Lecture 14
The Real Business Cycle Model
Part 1: Consumer

Hui-Jun Chen

National Tsing Hua University

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Overview

- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
  - Lecture 14: consumer
  - Lecture 15: firm
  - Lecture 16: competitive equilibrium
  - Lecture 17: formal example
  - Lecture 18: application to bring RBC to data

Real Business Cycle Model

- One of the workhorse frameworks in modern macroecnomics
- Real: not about *money* and *inflation*
- Business Cycle: mainly explain the short- and medium-term economics fluctuation ("business cycle frequency")
- Three agents: representative consumer, representative firm, and government
- All agents make **static and dynamic** decisions
- Larger "scale" model (i.e., more endogenous variables), but build upon the technique learned before

Outline

Consumer: Constraints

There are 11 variables associated with the representative consumer:

- choice variables: consumption (C, C') and labor supply (N_S, N_S')
  - leisure follows labor choice: $l = h - N_S$, and $l' = h - N_S'$
- owns the firm and get profits (π, π') and pays taxes (T, T')
- taken the equilibrium price as given (w, w', r)

Saving (S) at date 0 to construct lifetime budget constraint:

$$\begin{aligned} \text{today:} \quad & C + S = wN_S + \pi - T \\ \text{tomorrow:} \quad & C' = w'N_S' + \pi' - T' + (1 + r)S \end{aligned}$$

$$\text{lifetime constraint:} \quad C + \frac{C'}{1 + r} = \underbrace{wN_S + \pi - T}_{\approx Y \text{ in last lecture}} + \frac{\overbrace{w'N_S' + \pi' - T'}^{\approx Y' \text{ in last lecture}}}{1 + r}$$

Consumer: Preference

In general, utility fcn across consumption and labor choice can be mixed:

- e.g. mix C and N_S: GHH preferences
- e.g. mix current and future: Epstein – Zin preferences

Here, we are making simplified assumption: **additive for both direction**:

$$U(C, C', N_S, N_S') = u(C) - v(N_S) + u(C') - v(N_S'). \tag{1}$$

To see why additive can simplify analysis, recall the MRS in both intratemporal (w/i period) and intertemporal (b/w period) substitution:

$$MRS_{l,C} = -MRS_{N_S,C} = \frac{v'(N_S)}{u'(C)}, \text{ and } MRS_{C,C'} = \frac{u'(C)}{u'(C')}.$$

Representative Consumer's Problem

$$\begin{aligned} \max_{C, C', N_S, N_S'} \quad & u(C) - v(N_S) + u(C') - v(N_S') \\ \text{subject to} \quad & C + \frac{C'}{1+r} = wN_S + \pi - T + \frac{w'N_S' + \pi' - T'}{1+r}. \end{aligned} \tag{2}$$

- Hard to analyze in graph, $\because$ 4 choices variables $\Rightarrow$ 4-dim problem!
- Yet, usual procedure in Calculus still works!
- Why? Because partial derivatives only looks the optimality in 1-dim
- Each FOC is optimal for 1-dim $\Rightarrow$ solution satisfies ALL FOCs

Consumer's Optimality Conditions

- Step 1: substitute C by budget constraint,

$$\max_{C', N_S, N_S'} u\left(wN_S + \pi - T + \frac{w'N_S' + \pi' - T' - C'}{1+r}\right) - v(N_S) + u(C') - v(N_S')$$

- Step 2: find FOCs for C', N_S, and N_S':

$$\begin{aligned}
[C'] :&\quad u'(C') - \frac{1}{1+r}u'(C) = 0 \Rightarrow u'(C') = \frac{1}{1+r}u'(C) \\
[N_S] :&\quad wu'(C) - v'(N_S) = 0 \Rightarrow wu'(C) = v'(N_S) \\
[N_S'] :&\quad \frac{w'}{1+r}u'(C) - v'(N_S') = 0 \Rightarrow \frac{w'}{1+r}u'(C) = v'(N_S')
\end{aligned}$$

Consumer's Optimality Conditions (Cont.)

- Step 3: Compute multiple MRSs:

$$[C'] : \quad MRS_{C,C'} = \frac{u'(C)}{u'(C')} = 1 + r$$

$$[N_S] : \quad -MRS_{N_S,C} = MRS_{l,C} = \frac{v'(N_S)}{u'(C)} = w$$

$$[N_S'] : \quad MRS_{l',C} = \frac{v'(N_S')}{u'(C)} = \frac{w'}{1+r}$$

- Step 4: Get C by putting C', N_S and N_S' back to budget constraint.

Outline

Knowledge Gain from Consumer's Problem

We have derived 4 optimality conditions for 4 choice variables:

$$
\begin{aligned}
[C'] : &\quad MRS_{C,C'} = \frac{u'(C)}{u'(C')} = 1 + r \\
[N_S] : &\quad -MRS_{N_S,C} = MRS_{l,C} = \frac{v'(N_S)}{u'(C)} = w \\
[N_S'] : &\quad MRS_{l',C} = \frac{v'(N_S')}{u'(C)} = \frac{w'}{1+r} \\
\text{budget constraint} : &\quad C = wN_S + \pi - T + \frac{w'N_S' + \pi' - T' - C'}{1+r}
\end{aligned}
$$

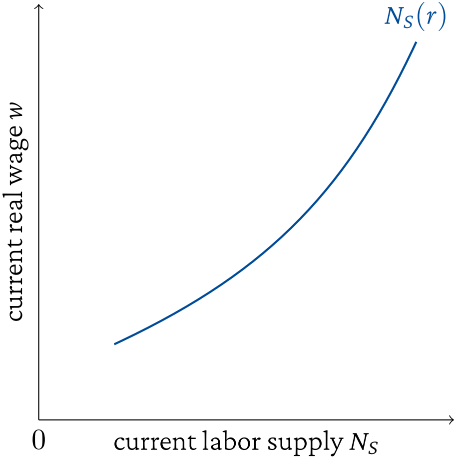
Recall that there are 11 variables, so still 7 variables remain. They are:

- 3 endogenous prices: w, w', r
- 4 endogenous quantities that shift lifetime wealth: π, π', T, T'

Need to know how consumer response to endogenous quantities!

Current Labor Supply and Current Wage

Figure: Figure 11.1 The Representative Consumer’ s Current Labor Supply Curve

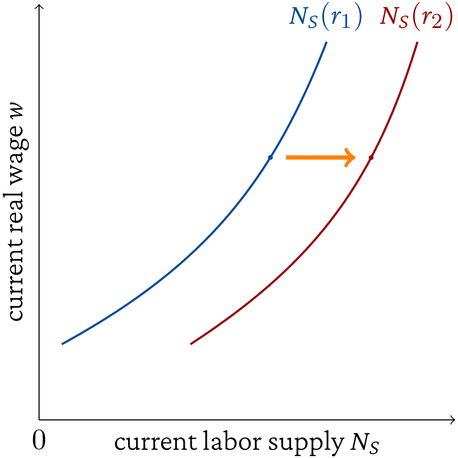


Assumption N1: current labor supply ↑ in current wage

- Recall two effects of wage on labor:
  - income (I): $l \uparrow, N_S \downarrow$
  - substitution (S): $l \downarrow, N_S \uparrow$
- **N1** suggests that substitution effect > income effect
- data: (I) and (S) cancel out in long-run, while RBC focus on short- and medium run!

Current Labor Supply and Real Interest Rate

Figure: Figure 11.2 Real Interest Rate ↑ Shifts the Current Labor Supply Curve to the Right



Assumption N2: current labor supply ↑ as real interest rate ↑

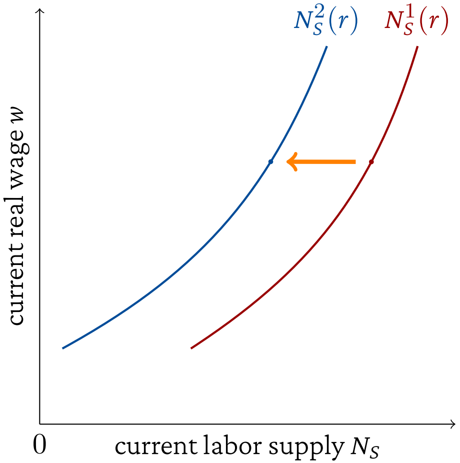
- can substitute intertemporally using **both** consumption and labor
- relative price of future leisure in terms of current leisure:

$$\frac{v'(N_S)}{v'(N_S')} = \frac{v'(N_S)}{u'(C)} \times \frac{u'(C)}{u'(C')} \times \frac{u'(C')}{v'(N_S')}$$
$$= w \times (1+r) \times \frac{1}{w'}$$

- fix w and w', $r \uparrow$ makes l become more costly, so $N_S \uparrow$

Current Labor Supply and Wealth

Figure: Figure 11.3 Effects of an Increase in Lifetime Wealth



Assumption N3: current labor supply $\downarrow$ as lifetime wealth $\uparrow$

- only pure income effect on normal goods (consumption & leisure), and thus labor decreases

Summary of Effect on Labor Supply

- **Assumption N1**: current labor supply $\uparrow$ in current wage
  - $\frac{dN_S}{dw} > 0$
- **Assumption N2**: current labor supply $\uparrow$ as real interest rate $\uparrow$
  - $\frac{dN_S}{dr} > 0$
- **Assumption N3**: current labor supply $\downarrow$ as lifetime wealth $\uparrow$
  - $\frac{dN_S}{dx} < 0$, where $x = \pi - T$.

All statements are properties about **supply curve**, not equilibrium quantities!