Lecture 15
The Real Business Cycle Model
Part 2: Firm

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Overview

- Recall that in Lecture 13, there is no production in dynamic model.
- The following 5 lectures is for **Real Business Cycle** (RBC) model:
  - Lecture 14: consumer
  - Lecture 15: firm
  - Lecture 16: competitive equilibrium
  - Lecture 17: formal example
  - Lecture 18: application to bring RBC to data

Outline

1 Demand for C

2 Representative Firm

3 More Assumptions

Demand for Consumption Goods

Ultimately, 3 markets will have to clear in the current period (date 0):

1. labor (like static model)
2. credit (like dynamic model)
3. consumption goods (implied in each case by Walras' Law)

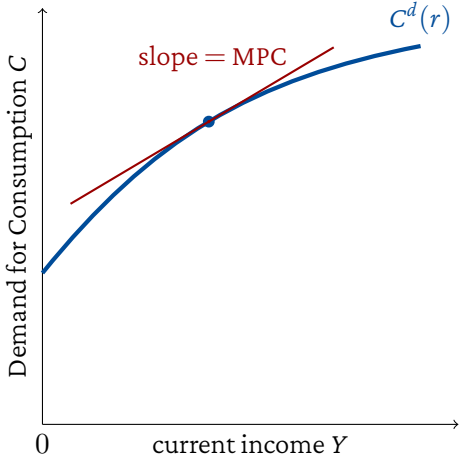
Recall our insights from last classes. Primary determinants of consumption:

- over lifetime: permanent income / lifetime wealth
- across periods: interest rate, current vs future income

Based on this, we'll construct a demand curve for current consumption goods that depends on lifetime wealth and the interest rate

Current Goods Demand and Current Income

Figure: Figure 11.4 Consumer's Current Demand for Consumption Goods Increases with Income

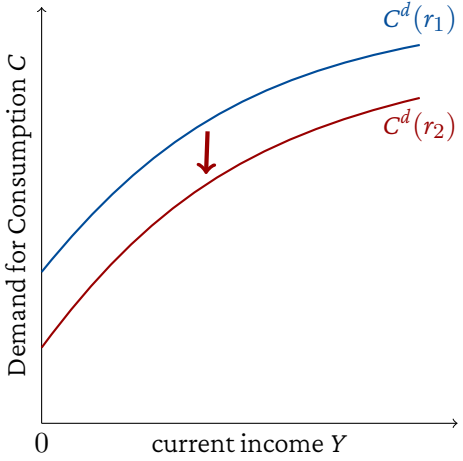


Assumption C1: demands for goods ↑ in income

- Recall pure income effect
- Slope of tangent line is **marginal propensity to consume** (MPC)
  - what fraction of Y ↑ goes to C?
  - $MPC = dC_D/dY$
- normal goods: both C and C' ↑, so saving S ↑
  - usually $MPC < 1$, i.e., not all Y ↑ goes to C.

Current Goods Demand and Real Interest Rate

Figure: Figure 11.5 Real Interest Rate ↑ Shifts the Demand for Consumption Goods Down

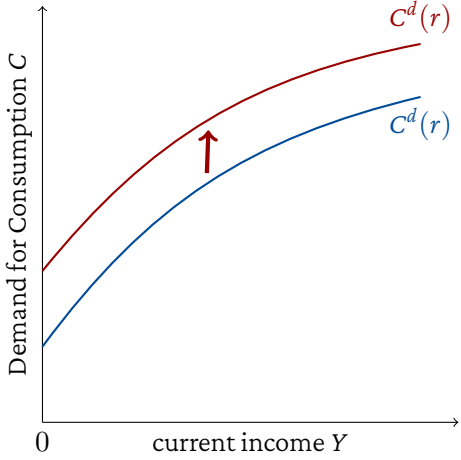


Assumption C2: demands for goods ↓ in real interest rate

- Recall both income and substitution effect (from dynamic model)
- Income effect: ambiguous (for borrowers and lenders)
- Substitution effect: always negative (for borrowers and lenders)
- **C2** assumes substitution effect dominates

Current Goods Demand and Lifetime Wealth

Figure: Figure 11.6 An Increase in Lifetime Wealth Shifts the Demand for Consumption Goods Up



Assumption C3: demands for goods ↑ in lifetime wealth

- similar to pure income effect

Note: consumer's demand is only one part of the GDP:

$$Y = C + I + G.$$

We'll discuss I and G in next lecture

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Overview: Firm Decision

- **production**: needs both capital K and labor N, $Y = zF(K, N)$
- **endowment**: firm is endowed with initial capital K
- **firm decision**:
  - both dates: labor (N), profit (π), and output (Y) by production
    $$Y = zF(K, N) \text{ and } Y' = z'F(K', N')$$
  - date 0 (today): **investment** (I) determines future capital K' given initial capital K and depreciation rate $\delta \in [0, 1]$,
    $$K' = (1 - \delta)K + I$$
- **Assumptions**:
  1. investment made in consumption goods
  2. remaining capital $(1 - \delta)K'$ liquidates tomorrow ($\because$ model ends)

Firm's Optimization Problem

Firm maximizes the discounted present value of profits:

$$\max_{N_D, N_D', K', I} \quad V = \pi + \frac{\pi'}{1+r} \quad \text{subject to} \quad K' = (1-\delta)K + I,$$

where $\pi = Y - wN - I$, and $\pi' = Y' - w'N' + \underbrace{(1-\delta)K'}_{\text{liquidate}}$.

Notice: since we assume that consumer owns the firm, so firm calculates present value using real interest rate r, i.e., how consumer discounts.

By substituting π, π', Y, Y' and I into above problem, we get

$$\begin{aligned} \max_{N_D, N_D', K'} \quad & zF(K, N_D) - wN_D - [K' - (1-\delta)K] \\ & + \frac{z'F(K', N_D') - w'N_D' + (1-\delta)K'}{1+r}. \end{aligned} \tag{1}$$

Firm's Optimality Conditions

$$
\begin{aligned}
[N_D]: &\quad zD_N F(K, N_D) = w \\
[N'_D]: &\quad z' D_N F(K', N'_D) = w' \\
[K']: &\quad -1 + \frac{z' D_{K'} F(K', N'_D) + (1-\delta)}{1+r} = 0
\end{aligned}
$$

- FOCs on current and future labor are the same as static model!
  - Why? Since labor choice is static: choose labor for current production
- FOC on future capital equalize the marginal cost and benefit of investment
  - cost: loss in current consumption (incurred today)
  - benefit: $\uparrow$ in marginal production + liquidating K' (incurred tomorrow)

Optimal Investment Schedule: Derivation

Solve for $[K']$, we get

$$z' D_{K'} F(K', N_D') + 1 - \delta = 1 + r \Rightarrow r = MPK' - \delta$$

For consumer, there are 2 assets to undertake intertemporal substitution:

1. saving in credit market (supply in credit mkt; demand in bond mkt)
2. capital held by the firm for production

Investing in capital means giving up (net) return r for (net) return $MPK' - \delta$: optimal investment rule means both must offset, WHY?

- if $r > MPK' - \delta$: consumer will save more for bond $\Rightarrow$ supply in credit market $\uparrow$, $r \downarrow$
- if $r < MPK' - \delta$: consumer asks firm to invest more capital $\Rightarrow MPK' \downarrow$

To sum up, $r = MPK' - \delta$ in equilibrium: "optimal" investment rule!

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Labor Demand is decreasing in w and increasing in z, K

Figure: Figure 11.7 The Demand Curve for N Is the Firm's MPL Schedule

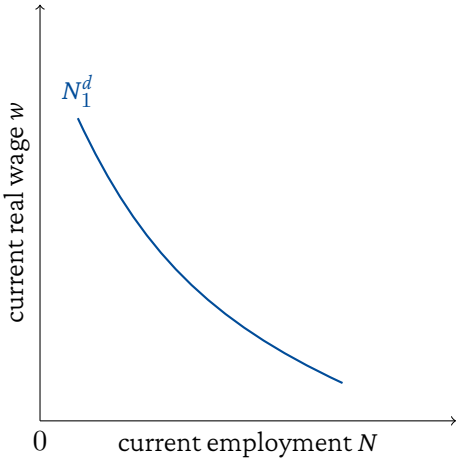
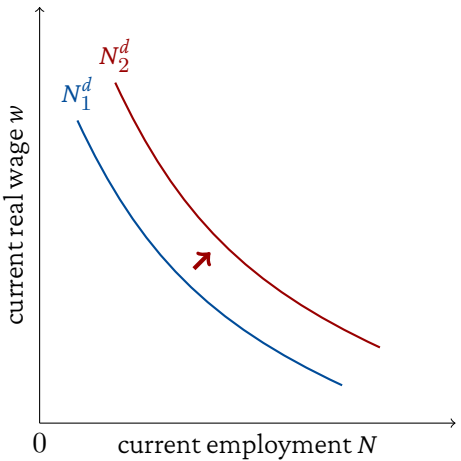
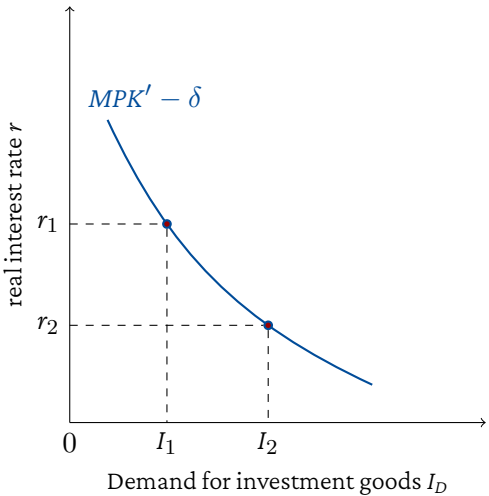


Figure: Figure 11.8 The Current Demand Curve for Labor Shifts Due to Changes in z and K



Optimal Investment Schedule: Graphical Representation

Figure 11.9 Optimal Investment Schedule for the Representative Firm

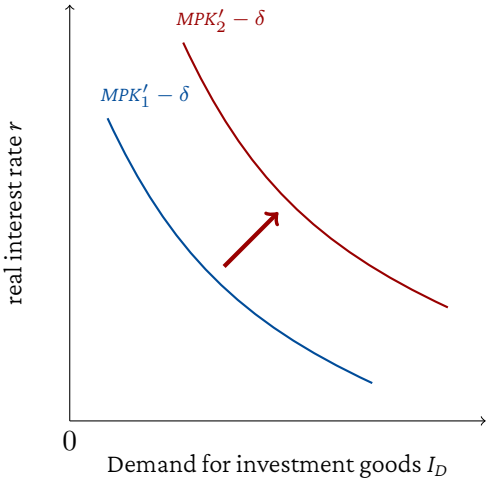


Put capital accumulation process into MPK and get
$z' D_{K'} F((1-\delta)K + I_D, N'_D) = r + \delta$

- as $r \uparrow$, need less K' for optimal investment schedule to hold.
  - why? diminishing MPK
- $K' \uparrow$ in I, so $r \uparrow$ also means less investment $\Rightarrow$ downward slope
- i.e., higher opportunity cost of investing

Optimal Investment Schedule: Effect of K and z'

Figure 11.10 The Optimal Investment Schedule Shifts to the Right if $K \downarrow$ or expecting $z' \uparrow$



The optimal investment schedule shifts to the right, i.e., demand for investment rises if

- current capital K decreases: $\frac{dI_D}{dK} < 0$
  - Intuition: need to invest more for less endowment
- (expected) future TFP increases: $\frac{dI_D}{dz'} > 0$
  - Intuition: investment is more productive