Macroeconomics II (Spring 2026)

Lecture 4: AD–AS in Equations

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Outline

From Lecture 3 to Lecture 4

- Lecture 3: AD–AS diagrams classify shocks by *comovement* of (Y, P).
- Lecture 4: write a compact set of equations that reproduces the same logic.
- Two objects will carry almost all intuition:
  - **Output gap** $x_t \equiv y_t - y_t^{FE}$ (tight vs slack economy)
  - **Expected price level** p_t^e (what firms/households had in mind when setting wages/prices)

Level vs rate (so we don't confuse P and π later)

- **Price level:** $p_t \equiv \log P_t$.
- **Inflation:** $\pi_t \equiv p_t - p_{t-1}$.
- A one-time increase in P_t is a level effect; persistent inflation means π_t stays high for several periods.
- In this lecture we keep the AD–AS vertical axis as P **(or** p**)** to match the figures.
- Next lecture we switch to π to study inflation dynamics explicitly.

Output gap and unemployment (connection only)

- We keep output-gap notation throughout: $x_t \equiv y_t - y_t^{FE}$.
- Labor market tightness is often summarized by the unemployment gap:

$$u_t - u_t^n \approx -\gamma\, x_t \qquad (\gamma > 0) \quad \text{(Okun's law)}.$$

- So:
  - $x_t > 0$ (output above potential) $\Rightarrow$ unemployment below natural
  - $x_t < 0$ (output below potential) $\Rightarrow$ unemployment above natural

Outline

Household Euler equation (RBC core)

- Representative household chooses $\{C_t, N_t, B_{t+1}\}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t)$$

subject to a budget constraint (bonds pay the real return $1 + r_t$).

- The intertemporal (bond) FOC is the Euler equation:

$$u_C(C_t, 1 - N_t) = \beta(1 + r_t)\, \mathbb{E}_t \left[u_C(C_{t+1}, 1 - N_{t+1}) \right].$$

Log utility Euler equation (no log-linearization language)

Assume **log utility** in consumption: $u(C) = \log C$, so $u_C(C) = 1/C$. The Euler equation becomes

$$\frac{1}{C_t} = \beta(1 + r_t)\,\mathbb{E}_t\left[\frac{1}{C_{t+1}}\right].$$

- If we ignore risk (certainty-equivalent intuition), this simplifies to

$$\frac{C_{t+1}}{C_t} = \beta(1 + r_t) \Rightarrow C_t = \frac{C_{t+1}}{\beta(1 + r_t)}.$$

- Higher r_t raises desired consumption growth: households postpone consumption $\Rightarrow$ current demand falls.

From the Euler intuition to a reduced-form IS

Holding expected future demand fixed, the Euler logic implies: **higher real rates reduce current demand.** A convenient gap-form summary (setting $\sigma = 1$ for log utility) is:

$$x_t = \mathbb{E}_t x_{t+1} - (r_t - r_t^n) + \varepsilon_t^d.$$

- r_t^n: the "natural" real rate consistent with $x_t = 0$.
- ε_t^d: demand shifters (fiscal, credit, sentiment).
- In AD–AS diagrams we often treat $\mathbb{E}_t x_{t+1}$ as roughly fixed within the period, giving a static IS.

From Euler to an IS equation in output-gap form

Use goods-market clearing and define the output gap:

$$x_t \equiv y_t - y_t^{FE}.$$

In sticky-price short-run analysis, output adjusts to demand, so consumption demand translates into output demand. Collect demand shifters into ε_t^d and define the **natural real rate** r_t^n as the rate that would make $x_t = 0$. A standard reduced-form IS relationship is:

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(r_t - r_t^n\right) + \varepsilon_t^d.$$

- ε_t^d: fiscal shocks, credit conditions, confidence, preference shifts.
- r_t^n: the "RBC" real rate consistent with potential output (no output gap).

Static approximation used in AD–AS diagrams

In the Lecture 3 AD–AS diagrams, we effectively treat expectations terms as fixed within the period. Holding $\mathbb{E}_t x_{t+1}$ constant gives a static IS:

$$x_t \approx -\tilde{\sigma}\left(r_t - r_t^n\right) + \varepsilon_t^d \qquad (\tilde{\sigma} > 0).$$

In levels this is the familiar linear form:

$$Y = \bar{A} - a\,r, \qquad a > 0.$$

- This is the IS curve you plug into IS–MP to derive a downward-sloping AD.

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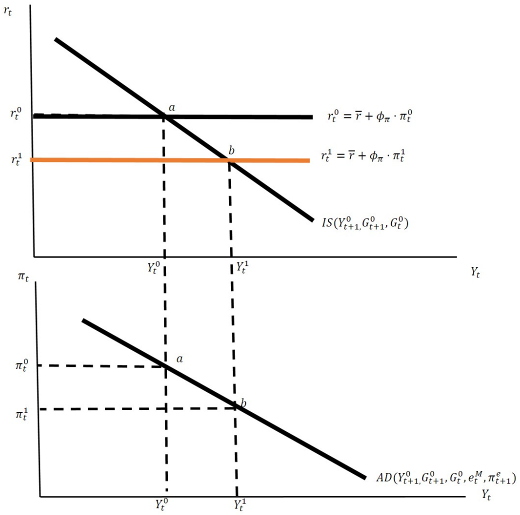
AD from IS–MP: the one mechanism



- Modern central banks adjust the **policy stance** in response to inflation and activity.
- To match the figure notation, we summarize the stance with a **real** policy rate r.
- Higher inflation (relative to target) $\Rightarrow$ higher r under the rule.
- Higher r reduces interest-sensitive spending $\Rightarrow$ lower output.
- Therefore higher P (through higher π) is associated with lower Y: **AD slopes down**.

A simple IS–MP system (linear, with r as the policy instrument)

- GDP Expenditure Accounting in this static model (no I_t):

$$Y_t = C_t(r_t) + G_t$$

  where current consumption is decreasing in r_t, while G_t is fixed.

- **IS (goods market):** output falls when the real rate rises, i.e., in a simplified form,

$$Y = \bar{A} - ar, \qquad a > 0,$$

  where $\bar{A}$ summarizes autonomous spending (preferences, G, taxes, etc.).

- **MP (policy rule in real-rate form):**

$$r = \bar{r} + \phi_\pi(\pi - \pi^*) + \phi_y(Y - Y^{FE}), \qquad \phi_\pi > 0,\ \phi_y \geq 0.$$

- **Link from prices to inflation:** $\pi \equiv p - p_{-1}$ with $p \equiv \log P$ and p_{-1} given within period t.

Solve IS + MP for $Y(\pi)$ (the AD equation)

Substitute the policy rule into IS:

$$Y = \bar{A} - a\Big[\bar{r} + \phi_\pi(\pi - \pi^*) + \phi_y(Y - Y^{FE})\Big].$$

Collect Y terms:

$$(1 + a\phi_y)Y = \bar{A} - a\bar{r} + a\phi_y Y^{FE} - a\phi_\pi(\pi - \pi^*).$$

- Therefore Y decreases in π: higher inflation $\Rightarrow$ higher r $\Rightarrow$ lower output.

From $Y(\pi)$ to $Y(P)$ (downward-sloping AD in (P, Y))

Use $\pi = p - p_{-1}$ with $p = \log P$ and p_{-1} predetermined. From the previous slide,

$$Y = \underbrace{\frac{\bar{A} - a\bar{r} + a\phi_y Y^{FE} + a\phi_\pi(p_{-1} + \pi^*)}{1 + a\phi_y}}_{\text{constant at time } t} - \underbrace{\frac{a\phi_\pi}{1 + a\phi_y}}_{\phi>0} p.$$

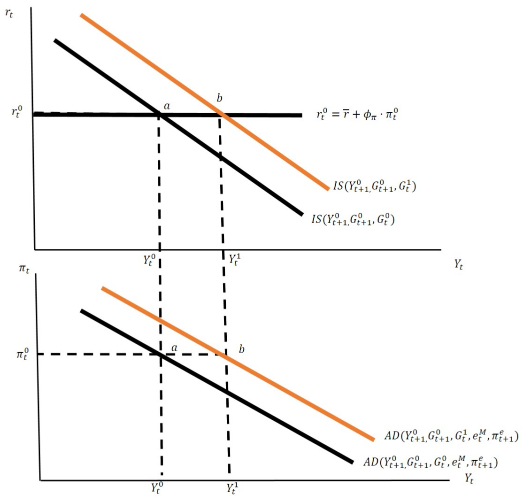
- Holding last period's price level p_{-1} fixed, higher p implies higher inflation π.
- The policy rule reacts by raising r, which lowers Y through IS.
- This reproduces the **downward slope of AD** in (P, Y) space without a money-supply LM curve.

What shifts AD (in equations)

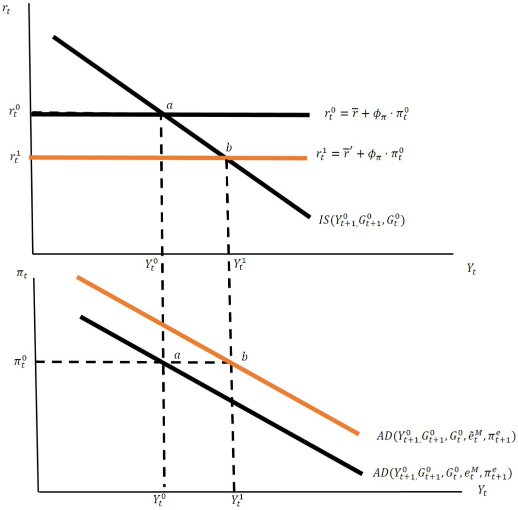
In this IS–MP derivation, AD shifts when the mapping from prices/inflation to output changes.

- **Demand shifters** ($\bar{A} \uparrow$): fiscal expansion, optimism, easier credit $\Rightarrow$ AD shifts right.
- **Policy stance** ($\bar{r} \downarrow$): more accommodative rule/intercept $\Rightarrow$ AD shifts right.
- **Potential output** ($Y^{FE} \uparrow$): raises output for given policy feedback.
- **Rule coefficients**:
  - larger ϕ_π makes AD *steeper* (policy leans harder against inflation),
  - larger ϕ_y dampens the response of Y to inflation (feedback through activity).

Two textbook shifts (same pictures, IS–MP interpretation)



$G \uparrow$ (or $\bar{A} \uparrow$): AD shifts right



Policy easing (lower $\overline{r}$ or a more accommodative rule): AD shifts right

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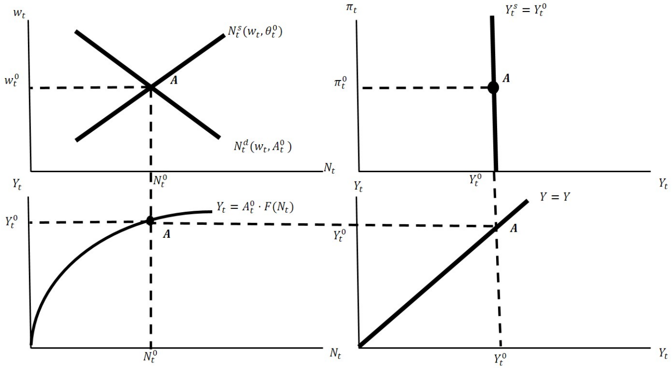
FE as the long-run supply benchmark



- FE output Y^{FE} comes from the RBC supply block (labor market + production).
- In the long run, prices adjust so output returns to Y^{FE}.
- FE shifts with technology, factor supply, and capital.

SRAS as "price surprises move output" (level form)

A convenient reduced form (Lucas/expectations-augmented supply):

$$y = y^{FE} + \alpha\,(p - p^e), \qquad \alpha > 0.$$

- If $p > p^e$ (prices higher than expected), real wages are lower than expected $\Rightarrow$ firms hire more $\Rightarrow$ output rises.
- If $p < p^e$, output falls below y^{FE}.
- The slope of SRAS in (p, y) space is $1/\alpha$.

SRAS written in output-gap notation

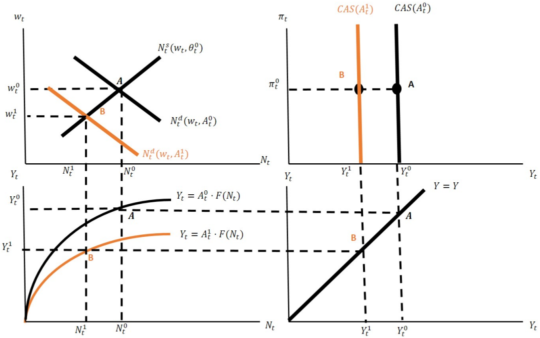
Define output gap $x \equiv y - y^{FE}$. Then

$$x = \alpha \left(p - p^e\right).$$

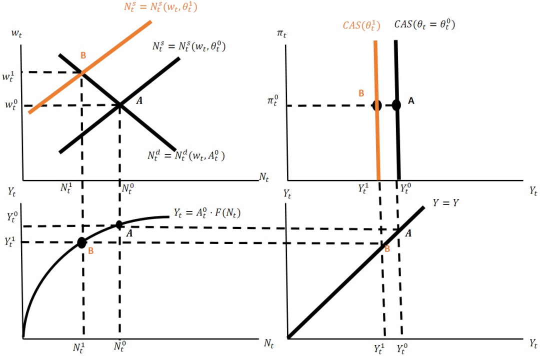
- Output gap is proportional to the **price-level surprise** $\left(p - p^e\right)$.

- A left shift of AS in Lecture 3 corresponds to shocks that raise marginal cost for given x, or (equivalently) raise the p needed to produce a given y.

- Next lecture we will rewrite this in inflation form and specify how p^e evolves (expected inflation).

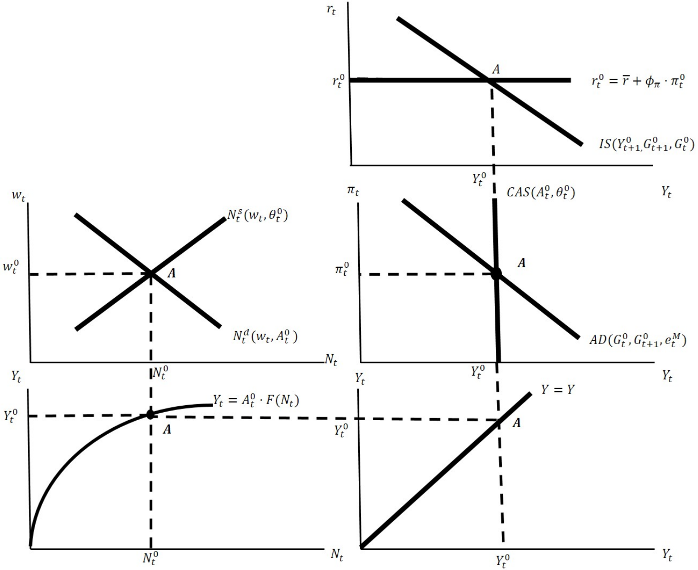


$A \downarrow$ shifts FE / CAS left



labor supply $\downarrow$ shifts FE / CAS left

Outline


r_t
r_t^0
A
$r_t^0 = \overline{r} + \phi_\pi \cdot \pi_t^0$
$IS(Y_{t+1}^0, G_{t+1}^0, G_t^0)$
Y_t^0
Y_t
π_t
$CAS(A_t^0, \theta_t^0)$
π_t^0
A
$AD(G_t^0, G_{t+1}^0, e_t^M)$
w_t
w_t^0
$N_t^s(w_t, \theta_t^0)$
$N_t^d(w_t, A_t^0)$
N_t
N_t^0
$Y_t = A_t^0 \cdot F(N_t)$
$Y = Y$

AD–AS equilibrium (solve once)

Use a simple AD reduced form $y = \bar{y}_D - \phi\, p$ with $\phi > 0$ (downward slope), and SRAS $p = p^e + \lambda(y - y^{FE})$ with $\lambda > 0$ (upward slope).

- Substitute AD into SRAS:

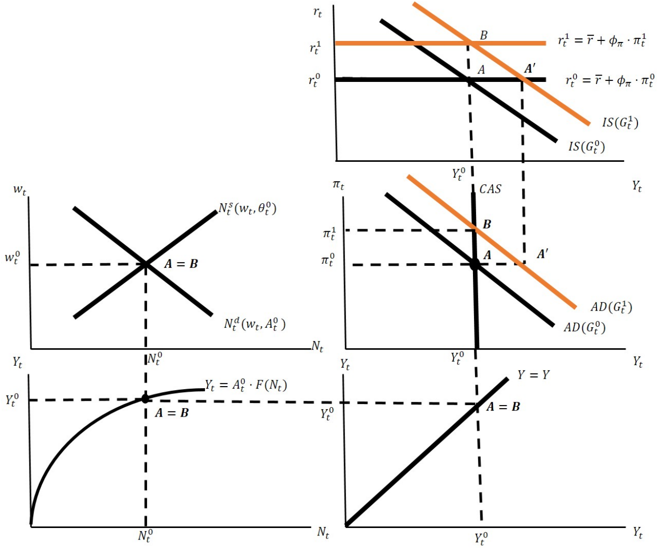
$$p = p^e + \lambda\big(\bar{y}_D - \phi p - y^{FE}\big).$$

- Solve for p:

$$(1 + \lambda\phi)p = p^e + \lambda(\bar{y}_D - y^{FE}) \quad \Rightarrow \quad p = \frac{p^e + \lambda(\bar{y}_D - y^{FE})}{1 + \lambda\phi}.$$

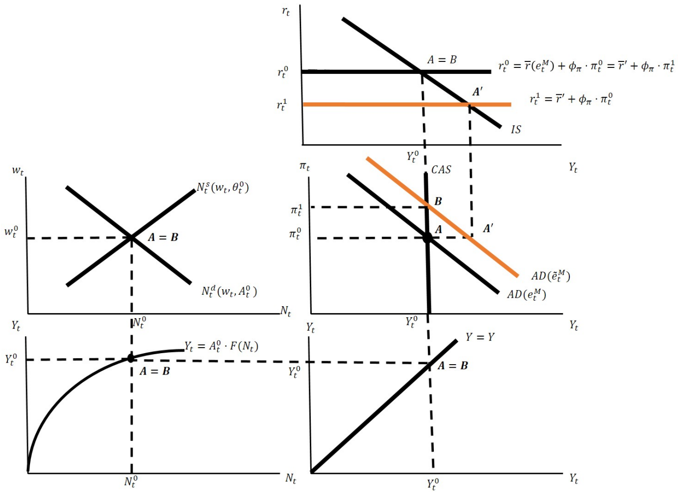
- Then y follows from AD: $y = \bar{y}_D - \phi p$.

Demand expansion: why (Y, P) move together



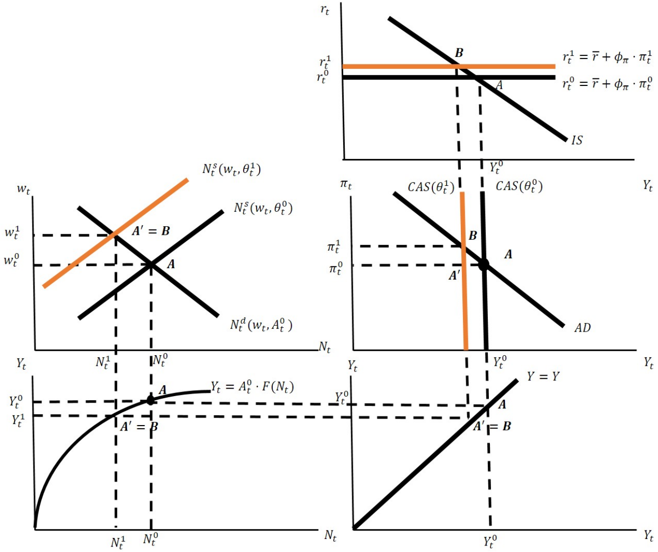
- Demand expansion means $\bar{y}_D \uparrow$ (AD shifts right).
- From the solved expression: p rises when $\bar{y}_D$ rises.
- AD then implies y rises as well.

Monetary easing: one parameter shift, same logic

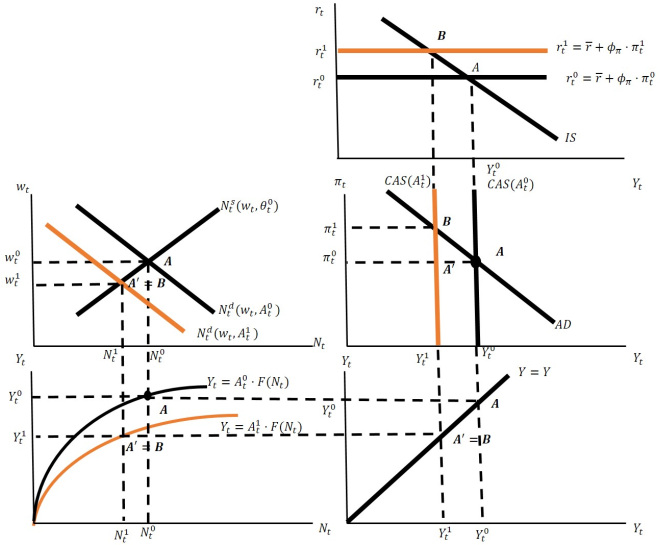


- Monetary easing shifts AD right (for each P, higher Y).
- In the IS–LM derivation, this is $M \uparrow$ or lower i (higher real balances / lower real rate).
- Equilibrium: (Y, P) rise together in the short run.

Supply contraction: stagflation in one line



- Supply contraction means $y^{FE} \downarrow$ or AS shifts up/left.
- In the solved expression: if y^{FE} falls, p rises.
- But lower y^{FE} also pushes output down.



- SRAS depends on p^e. When p^e updates, SRAS shifts.
- Medium-run logic: $p^e \uparrow$ until output returns to y^{FE}.
- This is the bridge to next lecture: specify how p^e evolves $\Rightarrow$ expected inflation.

Outline

Cheat sheet: what each symbol means in the math AD–AS

- **Demand side (AD):** $y = \bar{y}_D - \phi p$ (downward in p)
  - $\bar{y}_D$ collects *demand shifters*: G, taxes, wealth/optimism, monetary stance (M, i), etc.
- **Supply side (SRAS):** $p = p^e + \lambda(y - y^{FE})$ (upward in y)
  - y^{FE} is potential output from the RBC supply block (technology, labor, capital).
  - p^e is the expected price level; updating p^e shifts SRAS over time.
- **Unemployment link:** $u - u^n \approx -\gamma x$ (Okun).

Next lecture (without naming “Old Keynesian” yet)

- Replace p^e with a rule for expected inflation (how expectations are formed).
- Replace the level-form SRAS with an inflation form (Phillips curve language).
- Introduce central bank practice: inflation targeting, Taylor rules, and how a policy rule shifts AD.