Intermediate Macroeconomics II
Lecture 4: Old-Keynesian Inflation Dynamics

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From price to inflation

Price level P_t vs inflation π_t

- The AD–AS diagram in Lecture 3 told us how shocks move the **price level** P.

- Inflation is the **time-series object** we report and forecast.

$$\pi_t \equiv \log P_t - \log P_{t-1} \qquad \Rightarrow \qquad \log P_t = \log P_0 + \sum_{j=1}^{t} \pi_j.$$

- One-time *level* shift in P is different from a persistent run of $\pi_t > 0$.

- A model of inflation needs a **law of motion** for π_t (or P_t).

Output gap as "pressure"

- Full-employment (natural) output Y_t^{FE} comes from the RBC supply block.
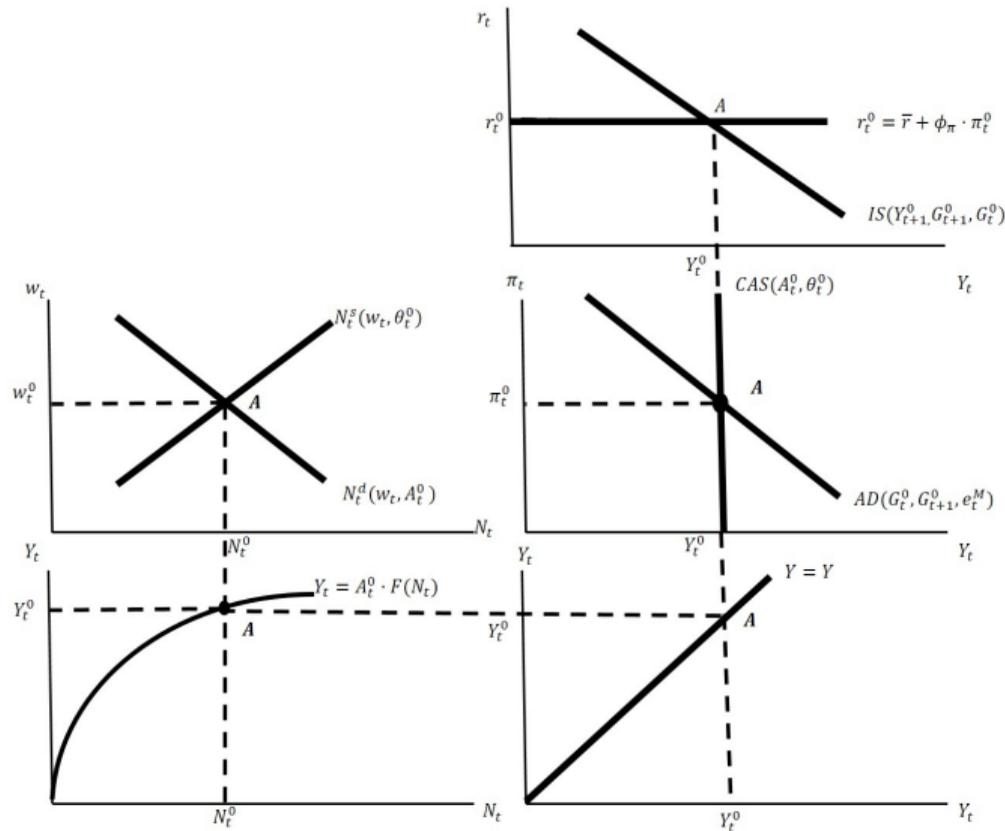
- Output gap (log):
$$x_t \equiv \log Y_t - \log Y_t^{FE}.$$

- Interpretation:
  - $x_t > 0$: tight factor markets, higher marginal cost pressure.

  - $x_t < 0$: slack, weaker marginal cost pressure.

A large share of inflation models can be read as: **inflation = expected inflation + κx_t + shocks**.

AD–AS recap (one picture to keep in mind)

- Lecture 3: **diagnosis** (demand vs supply) in (Y, P) space.
- This lecture: turn that diagnosis into **dynamics** in (x, π) space.

Key upgrade: specify how expected inflation is formed and how inflation adjusts over time.

Old-Keynesian inflation dynamics

Adaptive expectations (a benchmark)

$$\pi_t^e = \pi_{t-1}.$$

- Not "deep," but captures a common empirical feature: inflation inherits the recent past.

- It gives a simple way to connect **today's slack** to **tomorrow's inflation**.

- Later we replace this with rational/forward-looking expectations (NK).

Backward-looking Phillips curve (Old Keynesian)

$$\pi_t = \pi_{t-1} + \kappa x_t + u_t.$$

- $\kappa > 0$: stronger demand pressure ($x_t > 0$) makes inflation *accelerate*.

- u_t: cost-push shock (supply disruption) that can raise inflation even when x_t is small.

- This is the simplest way to turn "AS" into a **law of motion for inflation**.

IS/AD side in gap form (reduced form)

A convenient demand-side relation:
$$x_t = -\sigma \left(r_t - r_t^n \right) + \varepsilon_t^d,$$

where r_t^n is the natural real rate and ε_t^d is a demand disturbance.

- If the real rate rises above the natural rate, demand falls and the gap shrinks.

- This is the "IS" logic from Lecture 2, written in output-gap language.

Policy rule (replace LM with an interest-rate reaction)

$$i_t = \bar{i} + \phi_\pi(\pi_t - \pi^*) + \phi_x x_t.$$

- ϕ_π: response to inflation deviations.

- ϕ_x: response to the output gap.

- π^*: target inflation ($\approx 2\%$)

- Use Fisher: $r_t \approx i_t - \pi_t^e$.

This replaces "move M" with "move i" (modern institutional description).

Write deviations from a steady state with target inflation π^*:

$$\tilde{\pi}_t \equiv \pi_t - \pi^*, \qquad \tilde{i}_t \equiv i_t - i^*.$$

With adaptive expectations $\pi_t^e = \pi_{t-1}$ and Fisher $r_t \approx i_t - \pi_t^e$,

$$r_t - r^n \approx (i_t - i^*) - (\pi_{t-1} - \pi^*) = \tilde{i}_t - \tilde{\pi}_{t-1}.$$

The Taylor rule becomes

$$\tilde{i}_t = \phi_\pi \tilde{\pi}_t + \phi_x x_t \quad \Rightarrow \quad r_t - r^n = \phi_\pi \tilde{\pi}_t + \phi_x x_t - \tilde{\pi}_{t-1}.$$

Algebra 2: solve the IS equation for the output gap

Start from the reduced-form IS (demand) relation:

$$x_t = -\sigma(r_t - r^n) + \varepsilon_t^d.$$

Substitute $r_t - r^n = \phi_\pi \tilde{\pi}_t + \phi_x x_t - \tilde{\pi}_{t-1}$:

$$x_t = -\sigma(\phi_\pi \tilde{\pi}_t + \phi_x x_t - \tilde{\pi}_{t-1}) + \varepsilon_t^d.$$

Collect terms in x_t and solve:

$$(1 + \sigma\phi_x)x_t = -\sigma\phi_\pi \tilde{\pi}_t + \sigma\tilde{\pi}_{t-1} + \varepsilon_t^d \quad \Rightarrow \quad x_t = \frac{-\sigma\phi_\pi \tilde{\pi}_t + \sigma\tilde{\pi}_{t-1} + \varepsilon_t^d}{1 + \sigma\phi_x}.$$

Algebra 3: inflation becomes an AR(1) under policy feedback

Backward-looking Phillips curve:

$$\tilde{\pi}_t = \tilde{\pi}_{t-1} + \kappa x_t + u_t.$$

Plug in the solution for x_t:

$$\tilde{\pi}_t - \tilde{\pi}_{t-1} = \frac{\kappa}{1 + \sigma \phi_x}\Big(-\sigma \phi_\pi \tilde{\pi}_t + \sigma \tilde{\pi}_{t-1} + \varepsilon_t^d \Big) + u_t.$$

Rearrange:

$$\tilde{\pi}_t = \underbrace{\frac{1 + \sigma \phi_x + \kappa \sigma}{1 + \sigma \phi_x + \kappa \sigma \phi_\pi}}_{a} \tilde{\pi}_{t-1} + \underbrace{\frac{\kappa}{1 + \sigma \phi_x + \kappa \sigma \phi_\pi}}_{b} \varepsilon_t^d + \underbrace{\frac{1 + \sigma \phi_x}{1 + \sigma \phi_x + \kappa \sigma \phi_\pi}}_{c} u_t.$$

- Inflation persistence here is the coefficient a.

- Policy affects a through ϕ_π (and ϕ_x).

The Old-Keynesian core system (three equations)

1. **Demand (IS in gaps):** $\quad x_t = -\sigma(r_t - r_t^n) + \varepsilon_t^d$

2. **Inflation (backward-looking PC):** $\quad \pi_t = \pi_{t-1} + \kappa x_t + u_t$

3. **Policy (Taylor rule):** $\quad i_t = \bar{i} + \phi_\pi(\pi_t - \pi^*) + \phi_x x_t$

Together with $r_t \approx i_t - \pi_t^e$ and $\pi_t^e = \pi_{t-1}$, this system produces **inflation paths**, not just level shifts.

What the system implies

Why "raise rates to lower inflation" works here

Start from the Phillips curve:

$$\pi_t - \pi_{t-1} = \kappa x_t + u_t.$$

- To reduce inflation over time, the model needs $x_t < 0$ (slack) or $u_t < 0$.

- Tight policy raises r_t relative to $r_t^n \implies x_t$ falls (IS).

- Then inflation decelerates gradually because π_t inherits π_{t-1}.

Taylor principle, derived: why $\phi_\pi > 1$ stabilizes inflation here

From the algebra, inflation follows

$$\tilde{\pi}_t = a\,\tilde{\pi}_{t-1} + b\,\varepsilon_t^d + c\,u_t, \qquad a = \frac{1 + \sigma\phi_x + \kappa\sigma}{1 + \sigma\phi_x + \kappa\sigma\phi_\pi}.$$

- If $\phi_\pi > 1$, then $1 + \sigma\phi_x + \kappa\sigma\phi_\pi > 1 + \sigma\phi_x + \kappa\sigma \;\Rightarrow\; a < 1$ (mean reversion).

- If $\phi_\pi = 1$, then $a = 1$: inflation inherits a unit root from $\pi_t^e = \pi_{t-1}$.

- If $\phi_\pi < 1$, then $a > 1$: inflation tends to drift away after a shock.

This is the Old-Keynesian "doctrine" in algebra: reacting more than one-for-one raises the **real** rate when inflation rises, which pushes x_t negative and makes inflation decelerate.

A useful back-of-the-envelope: how much slack to disinflate?

From the Phillips curve,

$$\pi_t - \pi_{t-1} = \kappa x_t + u_t \quad \Rightarrow \quad x_t = \frac{\pi_t - \pi_{t-1} - u_t}{\kappa}.$$

- To reduce inflation by 1pp relative to last period (holding $u_t = 0$), you need

$$x_t \approx -\frac{1}{\kappa}.$$

- Smaller κ (flatter Phillips curve) means you need a larger output gap to change inflation.

- Cost-push shocks (u_t) can make disinflation harder even if demand is weak.

Demand-driven vs supply-driven inflation (same taxonomy as Lecture 3)

- **Demand inflation:** $\varepsilon_t^d > 0$ raises x_t.
  - Phillips: inflation accelerates through κx_t.

  - AD–AS: AD shifts right.

- **Cost-push inflation:** $u_t > 0$ raises inflation even if x_t is weak.
  - Phillips: direct inflation pressure.

  - AD–AS: AS shifts left.

A short narrative for disinflation

Suppose inflation is above target.

1. Policy raises i_t (rule or decision).

2. With π_t^e slow-moving, the real rate rises.

3. Demand weakens: x_t turns negative.

4. Inflation decelerates: $\pi_t - \pi_{t-1} = \kappa x_t + u_t$.

The key feature is **inertia**: inflation does not jump to target immediately; it moves as π_t updates from π_{t-1}.

Distinguishing theories

If the central bank chooses i_t, what pins down the price level?

- In the Old-Keynesian system, anchoring is effectively delegated to:

    - inertia in expected inflation ($\pi_t^e = \pi_{t-1}$), and

    - a sufficiently aggressive policy response ($\phi_\pi > 1$).

- Cochrane stresses that with rational expectations and modern institutions, the anchor can become fragile unless the full regime is specified.

Cochrane Ch.4: four theories, four different "anchors"

- **Money-growth:** money supply vs money demand determines inflation.

- **Old Keynesian:** inflation inherits the past; slack moves inflation gradually.

- **New Keynesian:** forward-looking pricing makes expectations and policy rules central.

- **Fiscal theory:** price level adjusts to value nominal government liabilities.

The same policy experiment (e.g., an interest-rate peg) can produce sharply different predictions across these theories.

Interest-rate peg thought experiment (why models disagree)

Hold i_t fixed for a long time.

- Old Keynesian: with $\pi_t^e = \pi_{t-1}$, inflation dynamics are pinned by inertia + slack adjustment.

- New Keynesian: with rational expectations, a peg can generate multiple inflation paths (indeterminacy) unless an anchor selects one.

- Fiscal theory: uniqueness can come from fiscal backing / debt valuation rather than from the interest-rate rule alone.

Preview: what changes next

Two ingredients change:

1. Replace $\pi_t^e = \pi_{t-1}$ with forward-looking expectations.

2. Replace the backward-looking Phillips curve with the NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t.$$

Then the policy rule does more than move demand: it influences inflation by shaping expected future inflation.

One-page map (where we are headed)

- Lecture 2: IS–LM $\Rightarrow$ AD.

- Lecture 3: AD–AS $\Rightarrow$ demand vs supply in (Y, P).

- Lecture 4: Old Keynesian $\Rightarrow$ inflation dynamics with inertia.

- Lecture 5+: New Keynesian $\Rightarrow$ expectations, determinacy, credibility.

- Later: fiscal regimes and the price-level anchor (FTPL).