RBC Model: Benchmark, Growth, and Measurement

Hui-Jun Chen

National Tsing Hua University
Department of Economics

Spring 2026

Outline

Overview: Real business cycle model

Goal: explain business-cycle fluctuations as efficient responses to real shocks (especially technology).

Core ingredients

- Representative household chooses $\{C_t, n_t, I_t, K_{t+1}\}$.
- Representative firm with CRS technology $Y_t = z_t F(K_t, n_t X_t)$.
- Perfect competition $\Rightarrow$ factors paid marginal products; profits are zero.
- No externalities/distortions $\Rightarrow$ welfare theorems apply.

Planner's problem (equivalent allocations):

$$\max_{\{n_t, C_t, K_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - n_t)$$

subject to

$$C_t \leq Y_t + (1-\delta)K_t - K_{t+1}, \qquad Y_t = z_t F(K_t, n_t X_t).$$

Representative household

$$\max_{\{n_t, C_t, I_t, K_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$$

subject to

$$C_t \leq w_t n_t + r_{k,t} K_t + \Pi_t - I_t,$$

$$K_{t+1} \leq (1-\delta) K_t + I_t, \qquad L_t \leq 1 - n_t.$$

- w_t: wage per hour, $r_{k,t}$: rental rate of capital.
- Π_t: firm profits (will be zero under CRS + competition).
- Household takes prices $\{w_t, r_{k,t}\}$ as given.

Representative firm

$$\max_{n_t, K_t} \left\{ Y_t - w_t n_t - r_{k,t} K_t \right\} \qquad \text{where} \qquad Y_t = z_t F(K_t, n_t X_t).$$

- F is CRS, strictly increasing, strictly concave in (K, nX).

- Perfect competition implies first-order conditions:

$$w_t = z_t \, D_{nX} F(K_t, n_t X_t) \cdot X_t, \qquad r_{k,t} = z_t \, D_K F(K_t, n_t X_t).$$

- CRS implies Euler's theorem $\Rightarrow$ profits are zero each period: $\Pi_t = 0$.

Markets and equilibrium logic

- Markets for Y, K, and n are perfectly competitive.
  - Factors paid marginal products.
  - CRS $\Rightarrow$ zero profits.
- No externalities or market imperfections.
  - Welfare theorems apply.
  - Equilibrium allocations can be recovered from the planning problem.
- Interpretation:
  - Competitive equilibrium pins down prices.
  - Planner representation is a shortcut to compute allocations.

Planner's problem (equivalent allocations)

$$\max_{\{n_t, C_t, K_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - n_t)$$

subject to

$$C_t \leq Y_t + (1-\delta)K_t - K_{t+1}, \qquad Y_t = z_t F(K_t, n_t X_t).$$

- u strictly increasing and strictly concave in $(C, 1-n)$.
- F CRS, strictly increasing and strictly concave in (K, nX).
- The planner chooses (C_t, n_t, K_{t+1}) directly using the economy's feasibility.

What RBC tries to explain (qualitative mechanism)

- Technology shocks z_t move productivity $\Rightarrow$ move the marginal products of labor and capital.
- Households respond by adjusting hours and saving/investment.
- Intertemporal smoothing $\Rightarrow$ consumption is smoother than output.
- Capital adjustment via investment $\Rightarrow$ investment is more volatile.

(Later) We will discipline z_t using the Solow residual and calibrate parameters to steady-state targets.

Outline

1. RBC model: household, firm, and planner
2. Exogenous technical progress and balanced growth
3. Growth-deflation (stationarizing the model)
4. Detrended stochastic RBC (indivisible labor) and DP viewpoint
5. Calibration and measurement (from data to shocks)

Exogenous technical progress (Harrod-neutral)

- Efficiency of each unit of labor is X_t.
  - If hours worked is n_t, then effective units of labor are $n_t X_t$.

- X_t is exogenous, non-stochastic, and grows at rate

$$\frac{X_{t+1}}{X_t} = \gamma_x \geq 1.$$

- Output is $Y_t = z_t F(K_t, n_t X_t)$ where F is CRS.

- When $\gamma_x > 1$, there is no steady state in levels. Instead there is long-run growth.

One-sector growth model (temporarily constant z)

We add labor-augmenting technical progress to the one-sector growth model.

$$\max_{\{C_t, n_t, I_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - n_t)$$

subject to

$$Y_t = zF(K_t, n_t X_t) \geq C_t + I_t,$$

$$(1-\delta)K_t + I_t \geq K_{t+1}, \qquad 0 \leq n_t \leq 1, \quad K_0 \text{ given.}$$

- For now, treat z as constant to focus on growth.

Balanced growth paths (BGP)

- Define the gross growth rate of variable v between dates t and $t+1$:

$$\gamma_v(t) \equiv \frac{v_{t+1}}{v_t}.$$

- Along a balanced growth path, all variables grow at constant rates:

$$\gamma_v(t) = \gamma_v \quad \text{(constant long-run growth rate).}$$

- BGP is the counterpart to a steady state when there is long-run growth.

Balanced growth 1: the resource constraint

Along the BGP, the resource constraint holds with equality:

$$Y_t = C_t + I_t.$$

At time 0 (WLOG), write $Y_0 \gamma_y^t = C_0 \gamma_c^t + I_0 \gamma_i^t$.

- For this to hold for all t, we must have
$$\gamma_y = \gamma_c = \gamma_i.$$
- So output, consumption, and investment grow at the same rate on the BGP.

Balanced growth 2: capital accumulation

Capital accumulation is the only constraint linking two dates:

$$K_{t+1} = (1-\delta)K_t + I_t.$$

Using growth rates, $K_0\gamma_k^{t+1} = (1-\delta)K_0\gamma_k^t + I_0\gamma_i^t$.

- For this to hold for all t, we need
$$\gamma_k = \gamma_i = \gamma_y.$$
- Combined with the resource constraint: $\gamma_k = \gamma_y = \gamma_c = \gamma_i$.
- Note: $n_t \in [0,1] \Rightarrow$ hours do **not** trend: $\gamma_n = 1$ on BGP.

Balanced growth 3: production

Production with labor-augmenting progress:

$$Y_t = zF(K_t, n_t X_t).$$

If F is CRS, write output per efficiency unit:

$$\frac{Y_t}{X_t} = zF\left(\frac{K_t}{X_t}, n_t\right).$$

- On BGP, n_t is constant and z is constant.
- For Y_t/X_t to be constant, we need K_t/X_t constant.
- Therefore, on BGP:

$$\gamma_y = \gamma_k = \gamma_x.$$

Balanced growth: prices (intuition)

On the BGP,

- The capital-output ratio K_t/Y_t is constant.

- The rental rate of capital in efficiency units is constant (no trend).

- The **real wage** per hour grows with labor efficiency:

$$w_t = (\text{wage per efficiency unit}) \times X_t \quad \Rightarrow \quad \gamma_w = \gamma_x.$$

Takeaway: growth comes from X_t; the stationary part (in efficiency units) has constant prices.

Balanced growth 4: preferences (when does BGP exist?)

For a BGP to exist, preferences must be compatible with scaling of consumption:

$$\beta^t u(C_t, 1-n_t) = \left(\beta\gamma_x^{1-\sigma}\right)^t u\left(\frac{C_t}{\gamma_x^t}, 1-n_t\right) \quad \text{for some constant } \sigma.$$

Otherwise, with C_t growing, marginal utilities change in a way that breaks stationarity.

KPR (1988): balanced growth requires either

$$u(C, 1-n) = \frac{C^{1-\sigma}}{1-\sigma} - v(1-n) \quad (\sigma > 0,\ \sigma \neq 1),$$

or

$$u(C, 1-n) = \log C + v(1-n).$$

Outline

Growth-deflated variables (stationarize the model)

- Economies are not always exactly on the BGP (transition dynamics; shocks).

- To study fluctuations, transform growing variables into efficiency units.

Define detrended (growth-deflated) variables:

$$y_t \equiv \frac{Y_t}{X_t}, \quad c_t \equiv \frac{C_t}{X_t}, \quad i_t \equiv \frac{I_t}{X_t}, \quad k_t \equiv \frac{K_t}{X_t}.$$

- Divide each constraint by X_t.

- Watch for γ_x in capital accumulation (and Euler).

Growth-deflated planning problem

$$\max_{\{c_t, n_t, i_t, k_{t+1}\}} \sum_{t=0}^{\infty} (\beta^*)^t \, \tilde{u}(c_t, 1 - n_t)$$

subject to

$$y_t = z_t F(k_t, n_t), \qquad y_t \geq c_t + i_t,$$

$$\gamma_x k_{t+1} \leq (1 - \delta) k_t + i_t, \qquad 0 \leq n_t \leq 1, \quad k_0 \text{ given.}$$

- For CRRA: $\beta^* = \beta \gamma_x^{1-\sigma}$. For log utility ($\sigma = 1$), $\beta^* = \beta$.
- Now the problem is stationary in (k, z).

Outline

An RBC model with indivisible labor (detrended DP problem)

Using employment lotteries (Rogerson, 1988) / Hansen (1985), we can think of the problem as a representative household.

A common specification (detrended):

$$u(c, n) = \log c - Bn, \qquad B > 0.$$

Let productivity z follow a Markov chain with states $\{z_j\}_{j=1}^{N_z}$ and transition $\pi_{ij} = \Pr(z' = z_j \mid z = z_i)$.

Bellman equation (state (z, k)):

$$V(z, k) = \max_{c,n,k' \geq 0} \left\{ \log c - Bn + \beta \sum_{j=1}^{N_z} \pi_{ij} V(z_j, k') \right\}$$

subject to

$$c + \gamma_x k' - (1-\delta)k \leq zF(k, n), \qquad z, k \text{ given.}$$

Decision rules (policy functions)

Solving the detrended RBC DP yields policy functions:

$$c = c(z, k), \qquad n = n(z, k), \qquad k' = g(z, k).$$

- **State:** current productivity z and detrended capital k.
- **Choices:** consumption c, hours n, next capital k'.
- **Equilibrium aggregation:** representative-agent allocation equals competitive equilibrium allocation.

Computational note: later we can solve this using DP methods (VFI / policy iteration), without linearization.

Outline

1. RBC model: household, firm, and planner
2. Exogenous technical progress and balanced growth
3. Growth-deflation (stationarizing the model)
4. Detrended stochastic RBC (indivisible labor) and DP viewpoint
5. Calibration and measurement (from data to shocks)

Steady state relationships (for calibration)

For Cobb–Douglas $F(k,n) = k^{\alpha} n^{1-\alpha}$ and detrended steady state ($z = 1$, constants):

$$y^* = (k^*)^{\alpha}(n^*)^{1-\alpha}.$$

Growth-deflated capital accumulation on BGP:

$$i^* = (\gamma_x - 1 + \delta)k^* \quad \Rightarrow \quad \frac{i^*}{k^*} = \gamma_x - 1 + \delta.$$

Euler (no co-state notation):

$$1 = \beta\left(\alpha\frac{y^*}{k^*} + 1 - \delta\right)\frac{1}{\gamma_x} \quad \Rightarrow \quad \beta = \frac{\gamma_x}{\alpha\frac{y^*}{k^*} + 1 - \delta}.$$

Intratemporal (for $u = \log c - Bn$): $B = \frac{w^*}{c^*}$ and $w^* = (1-\alpha)\frac{y^*}{n^*}$, so

$$Bn^* = \frac{y^*}{c^*}(1-\alpha).$$

Calibration targets (typical quarterly)

Targets (examples from the source notes)

- Annual GDP growth $\approx 1.6\% \Rightarrow$ quarterly $\gamma_x \approx 1.004$.
- Annual real interest rate $\approx 4\% \Rightarrow$ quarterly $r \approx 1\%$.
- Annual $I/K \approx 0.07 \Rightarrow$ quarterly $i/k \approx 0.0175$.
- Labor share $\frac{wn}{y} \approx 0.66 \Rightarrow \alpha \approx 0.34$.
- Average fraction of time worked: $n^* \approx 1/3$.

Use steady-state equations to pin down β, δ, α, B given these targets.

Measuring capital: perpetual inventory method

Capital accumulation:

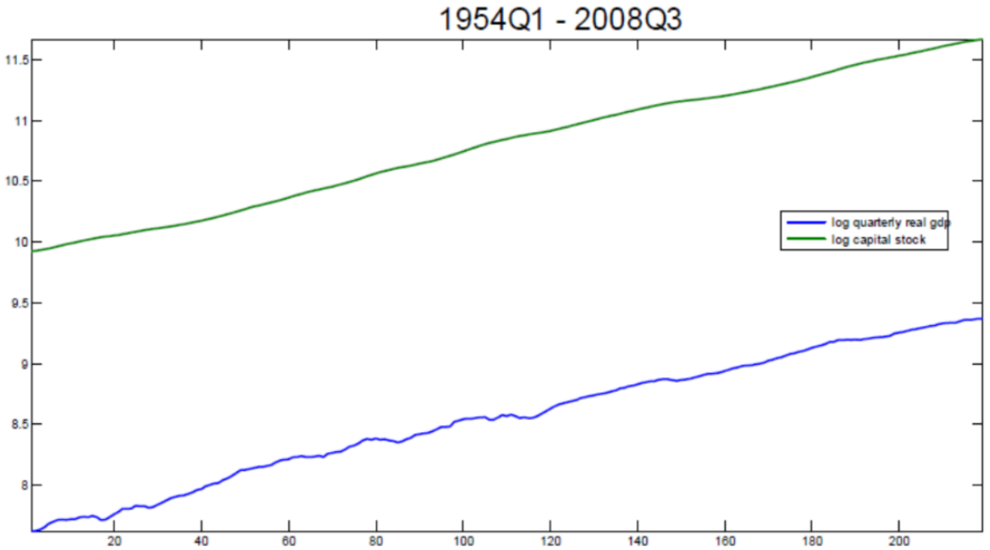
$$K_{t+1} = (1-\delta)K_t + I_t.$$

Iterate forward:

$$K_t = (1-\delta)^t K_0 + \sum_{s=0}^{t-1} (1-\delta)^s I_{t-s-1}.$$

- If K_0 is unknown, impose long-run growth restrictions to infer K_0.
- This is the standard way to construct a capital stock series from investment data.

Measuring capital: GDP vs capital stock (illustration)



Measuring the Solow residual (TFP)

Gather data on $\{Y_t, K_t, n_t\}$. With Cobb–Douglas production,

$$Y_t = A_t K_t^{\alpha} (n_t X_t)^{1-\alpha}.$$

Taking logs:

$$\log A_t = \log Y_t - \alpha \log K_t - (1-\alpha)\log n_t - (1-\alpha)\log X_t.$$

Decompose A_t into deterministic growth and stationary technology:

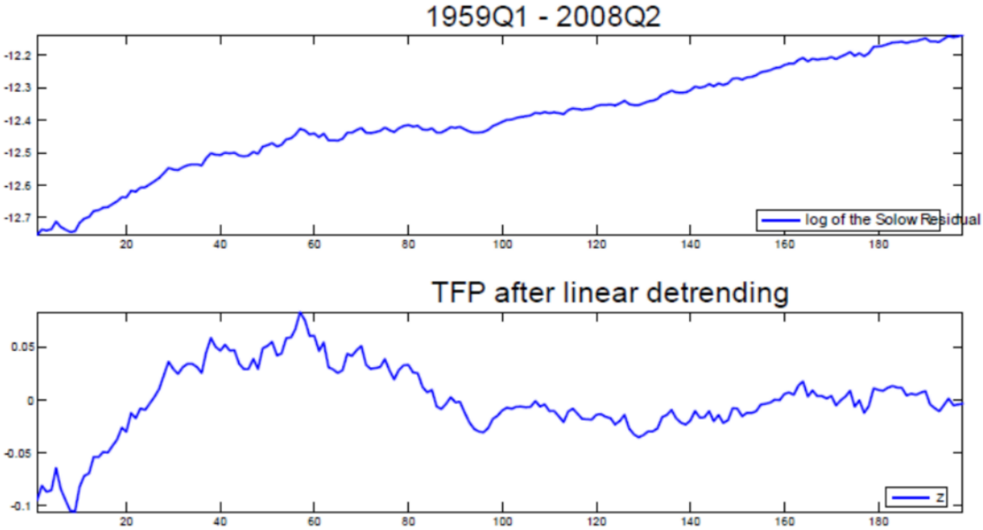
$$A_t = z_t X_t^{1-\alpha}, \qquad \frac{X_{t+1}}{X_t} = \gamma_x.$$

Then a common specification is an AR(1) for the stationary component:

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1}, \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2).$$

Estimating $(\rho, \sigma_\varepsilon)$ pins down the persistence and variance of technology shocks.

Solow residual plot (illustration)



Wrap-up: RBC pipeline

- Start with the benchmark RBC model (household + firm + markets).
- Introduce growth (labor-augmenting X_t) $\Rightarrow$ BGP rather than steady state.
- Growth-deflate to get a stationary problem in efficiency units.
- Measure K_t and z_t from data (perpetual inventory, Solow residual).
- Use calibrated parameters and estimated shocks to study business-cycle dynamics.