Debt Financing, Used Capital Market and Capital Reallocation

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Abstract

I study how much financial frictions and the endogeneity of partially irreversible capital explain the slow recovery of the Great Recession. I propose a heterogeneous firm model with real and financial frictions. Firms adjust their capital stock by trading on the used capital market; thus, the capital partial irreversibility is endogenized by the price in the market. This irreversibility creates two opposite forces affecting investment volatility. First, capital investment is relatively cheaper in the recession, and thus attracts firms to invest in capital, dampening the fall of aggregate investment. Second, in a downturn, the capital becomes less reversible, and investments become riskier, exacerbating the fall of aggregate investment. In my model, the collateral constraint is procyclical since it is based on the resale value of the capital, and thus amplifies the first force and dampens the response of aggregate investment. I found that in the steady state, the used capital market induces firms to stay financially constrained due to lower effective capital prices. This status however may put these firms in a vulnerable position when the value of their collateral drops during a recession, as they heavily depend on debt to finance their capital investment. However, the perfect foresight exercise shows that the time-varying collateral constraint channel is relatively small. The main channel lies in the used capital market price, which acts as an automatic stabilizer during the credit crisis.

Keywords: Investment irreversibility, Collateral Constraint, Business Cycle

JEL Codes: E22, E23, E32, G31

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1 Introduction

How does debt financing interact with capital reallocation when capital irreversibility is endogenized? Firms trade used investments in large quantities both directly through the secondary market of equipment or indirectly through acquisitions. Meanwhile, the Great Recession, which originated from the failure of both the real and financial sectors of the United States, has unusually slow recovery in investment and employment since the trough in $Q2$ of 2009. My goal is to quantitatively characterize the relationship between debt financing and capital reallocation using a dynamic stochastic general equilibrium model to explain the slow recovery of the Great Recession.

The model is designed to match several stylized facts in the empirical literature: (a) In the market of used investment, smaller firms are usually the buyers, while larger firms are usually the sellers (Eisfeldt and Rampini, 2006, 2007), (b) debt financing is positively correlated with capital turnover (Eisfeldt and Shi, 2018), and (c) capital turnover is procyclical, while its measured benefit[1] is either acyclical or countercyclical. I propose a heterogeneous firm model with capital partial irreversibility being endogenized by the used investment price, and borrowing limit based on the market evaluation of the collateral. This model matches the stylized facts by introducing two opposing forces: In the recession, firms' sales per unit of capital decrease, causing the capital turnover to drop. As the market evaluation of capital is shrinking in the foreseeable future, firm borrowing ability is restricted, and so does the capital stock holding. Thus, the dispersion of capital productivity decreases. On the other hand, the price of the used investment also decreases, creating the incentive for smaller firms to purchase the used investment, which can potentially result in a higher standard deviation in the dispersion of capital productivity. As a result of two opposite forces, the measured benefit of capital turnover is either acyclical or countercyclical, leading to the slow recovery of the Great Recession.

The model has three distinct features. First, it incorporates two frictions regarding capital reallocation: endogenous capital irreversibility as real friction, and collateral constraint as financial friction. Capital irreversibilities in investment allow firms to follow *(S, s)* policy in terms of their capital decision. The collateral constraint provides a clear distinction between small and large firms as financially constrained and unconstrained, respectively. As I study how firms' financial decision interacts with their capital allocation, both frictions are indispensable for my study. Second, the degree of capital irreversibility is endogenized by

[1] The benefit of capital turnover is usually measured by the dispersion in the productivity of capital across the firm. The measurements include the cross-sectional standard deviation of Tobin's q, TFP growth rates dispersion, and capacity utilization dispersion. See Eisfeldt and Rampini (2006) for the initial result, and Eisfeldt and Shi (2018) for the updated result.

the price in the used investment market. Following the specification of Lanteri (2018), the used investment price is determined by the supply and demand of this market. The supply comes from the large firms that want to downward adjust their capital stock, while the small firms which upward adjust their capital demand the used investment. The resulting inaction region will be enlarged during the recession since smaller firms are more prone to aggregate fluctuation. I suppose that the financial friction will generate a persistent inaction region, and create a slow recovery as the price in the used investment market slowly adjusts. Third, the collateral constraints in my model are a fraction of the market value of the capital stocks owned by the firm, which creates a direct connection between the used investment price and financial friction. The lenders don't care about the stock of the collateral; they care about the market value of the collateral in case the borrowers cannot repay. This directly links the aggregate fluctuation in real sectors to the financial sectors, generating large credit restrictions in the downturn. I found out that in the steady state, there are more firms willing to stay in financially constrained status due to the cheaper effective capital price. Nevertheless, remaining in debt may put firms in a vulnerable position due to the credit restriction they will face during the recession. This will exacerbate the responsiveness of aggregate investment, leading to a slow recovery of the economy.

My model is related to several strands of literature. First, my work is related to the empirical literature analyzing capital reallocation. Ramey and Shapiro (2001) use the data of closing aerospace plants to report the resale prices of their physical assets. They conclude that there exists significant capital irreversibility in different sectors and adjustment costs to install the used investment. Eisfeldt and Rampini (2006) use Compustat data to show that the Sales of property, plant, and equipment (PP&E), and acquisitions are procyclical, while its measured benefit is countercyclical. Eisfeldt and Shi (2018) summarized the empirical findings from Eisfeldt and Rampini (2006) and provided an up-to-date analysis with respect to those in Eisfeldt and Rampini (2006). They also provide a one-period general equilibrium model featuring collateral constraints and liquidity cost or adjustment cost. Eisfeldt and Rampini (2007) use data from Vehicle Inventory and Use Survey (VIUS) and Annual Capital Expenditures Survey (ACES) to show that financially constrained firms are more likely to purchase used investments and operate on a smaller scale. Edgerton (2011) uses three different datasets to conclude that the investment tax credit has a significant and large effect on the relative price of used farm machinery and similar but less robust result in the aircraft industry. He uses the two existing datasets on sales of aircraft and farm machinery and one assembled dataset on the auction sales of used construction machines from the Internet. All the above empirical literature suggests that capital irreversibility interaction with financial frictions and different sectors can have different degrees of irreversibility, which would be the interest to future research.

Second, my model is deeply connected with a large literature on the effect of financial friction on the real sectors. Kiyotaki and Moore (1997) introduce the collateral constraint and construct a model of credit cycles to emphasize the importance of collateral constraints. On the household side, Boz and Mendoza (2014) study the effect of credit constraints on a representative household using land as collateral. They assume Bayesian learning and explain substantial increases in debt and the price of the land given optimal priors. Gavazza and Lanteri (2021) develop a heterogeneous households model to investigate the effect of collateral constraints on households' decisions in trading or scraping their durables. On the firm side, Arellano et al. (2019) show that uncertainty shocks generate worsened credit conditions. Lanteri and Rampini (2023) characterize the efficiency of a heterogeneous firm model with collateral constraint and proposed collateral and distributive externalities.

Two papers, both heterogeneous firm models with capital irreversibility, are the foundation of my work. Lanteri (2018) is the first paper to develop a general equilibrium model in which firms replace their capital by trading in the secondary market, and thus endogenized the capital irreversibility. He proposes a tractable mechanism to incorporate the usage of new and used investments into a CES aggregator. This method avoids the necessity to track two assets but one when solving the optimization problem. Khan and Thomas (2013) is the first paper to explore the effect of endogenous total factor productivity (TFP) shocks in a dynamic stochastic general equilibrium setting. They show that credit shock can generate a large and persistent recession due to the change in the distribution of firms. Their model formulation and numerical method used to solve the model provide me with tools to incorporate collateral constraints into the model environment specified by Lanteri (2018).

There are several well-known approaches to financial frictions other than collateral constraints. Cooley et al. (2004) develop a general equilibrium model with limited enforceability of contract and conclude that the lower the enforceability is, the higher the macroeconomic volatility can be. Jermann and Quadrini (2012) proposed a representative firm model with the enforcement constraint, which evolves from the limited enforceability.

The rest of the paper is described as follows. Section 2 presents the model environment in detail, Section 3 analyzes the model environment to utilize the numerical algorithm in Khan and Thomas (2013), Section 4 shows the calibration target and how well my model matches them, Section 5 presents the results, and Section 6 concludes.

2 Model

My model is a heterogeneous firm model with the used capital market and collateral constraints. I begin describing the model economy by describing the optimization problem faced by firms, followed up with the formulation of the household problem, as well as the definition of the recursive equilibrium.

2.1 Firms

I assume a continuum of firms between $[0,1]$ in which they produce homogeneous goods using predetermined capital stock k and labor n. The production function is $y = z\varepsilon F(k,n) = z\varepsilon k^{\alpha}n^{\nu}$, where $\alpha+\nu < 1$ exhibits decreasing return to scale. At the beginning of each period, I assume that π_d fraction of firms are forced to exit to prevent firms from accumulating enough resources such that no firm is subject to the borrowing constraints. In each period, the firm is facing two exogenous shocks that follow a Markov chain: (a) an idiosyncratic productivity shock $\varepsilon \in \{\varepsilon_1, \dots \varepsilon_{N_\varepsilon}\}$, where $\Pr(\varepsilon' = \varepsilon_j | \varepsilon = \varepsilon_i) = \pi^{\varepsilon}_{ij}$, and (b) an aggregate TFP shock $z \in \{z_1, \dots z_{N_z}\}$, where $\Pr(z' = z_g | z = z_f) = \pi^{z}_{fg}$. A firm is defined by (1) its amount of capital stock k, (2) its level of one-period debt holding b borrowed or lent from the household, and (3) its current idiosyncratic productivity realization ε. For simplicity, I denote firm-level state variables $\mathbf{s} \equiv \{k, b, \varepsilon\}$ and aggregate state variables z. Given the realization of firm-level state $\mathbf{s}$ and aggregate state z, the firm maximizes the expected discounted value function by choosing current employment n, next-period capital level k', and next-period debt holding b'. For each labor unit firm demanded, it pays wage bill w, which is subjected to the aggregate state z. For each debt unit it borrows for the next period, the firm repays q_b unit of output, and thus the relative bond price q_b^{-1} reflects the interest rate of this economy. For the decision on each unit of next-period capital, the adjustment process follows the specification of Lanteri (2018). If a firm invests in a nonnegative level, i.e., the firm is expanding, then its capital accumulation process is

$$\begin{aligned} k' &= (1-\delta)k + I(i_{new}, i_{used}) \\ I(i_{new}, i_{used}) &= \left[\eta^{\frac{1}{s}}(i_{new})^{\frac{s-1}{s}} + (1-\eta)^{\frac{1}{s}}(i_{used})^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}, \end{aligned} \tag{1}$$

where i_{new} and i_{used} are the new and used investment in aggregated by the $I(\cdot)$ function. $I(\cdot)$ is the constant elasticity of substitution (CES) aggregator between new and used investments. Inside this CES aggregator, the parameter $\eta \in (0,1)$ determines the average ratio between new and used investment, and $\varepsilon > 0$ represents the elasticity of substitution between new and used investment. The specification of η and s ensures that the composition $I(\cdot)$ is a constant return to scale technology in both i_{new} and i_{used}. The corresponding price index associated with the composition $I(\cdot)$ is

$$Q = \left[\eta + (1-\eta)(q+\gamma)^{1-s}\right]^{\frac{1}{1-s}}, \tag{2}$$

where the cost of a unit of new capital in terms of units of output is normalized to 1, and the counterpart of used investment is $q+\gamma$. q is the trading price of used investment, and γ is the per-unit reallocation cost. In any equilibrium with positive trading volume in used investment, the used investment price combined with the reallocation cost must be cheaper than the new investment price, and thus $q+\gamma \leq 1$ results in $q \leq 1-\gamma$. Following Lanteri (2018), I assume the strict inequality holds in equilibrium, i.e., $q < 1-\gamma$ leads to $Q < 1$.

My model economy consists of both real and financial friction. When firms are adjusting their capital stocks, they are subject to the real frictions that are caused by the price fluctuation of the used capital market. To be specific, if the firm is expanding, i.e., investing in capital, $i \geq 0$, its capital follows the law of motion for upward-adjusting firm, $Qk' = Q(1-\delta)k + i$, where $\delta \in (0,1)$ is the rate of depreciation. On the other hand, for a downward-adjusting firm, the firm is disinvesting, $d \geq 0$, and its capital accumulation formula is $qk' = q(1-\delta)k - d$. Notice that the existence of investment selling price q and investment purchasing price Q, and since $q < Q$ by construction, the price in the used capital market endogenized the capital irreversibility. The financial friction for firms is that their borrowing ability is limited by their capital stock. They can only borrow one-period bonds up to a fraction of their capital stock. To be specific, $b' \leq q\zeta k$. Firms' current investment decisions may also be affected by their decision on next-period bond holding. Thus, with both real and financial frictions, I must treat both capital and bond as distinct state variables.

The distribution of firms μ over (k, b, ε) is an endogenous aggregate state that follows the evolution mapping Γ from its current state, i.e., $\mu' = \Gamma(z, \mu)$. As mentioned before, a fraction of π_d of firms will exit the economy and be replaced by newborns every period. Following Khan and Thomas (2013), I assume that the entering firms (1) bears no debt, (2) endowed with idiosyncratic productivity ε_0,which are drawn from ergodic distribution implied by $\{\pi_{ij}^{\varepsilon}\}$ and (3) endowed with an initial capital stock k_0, which is determined by χ fraction of the long-run aggregate capital stock.

I now start illustrating the problem solved by each agent in this economy. Let $v_0(k, b, \varepsilon; z, \mu)$ be the expected discounted value of a firm with (k, b) asset holding and ε realization of productivity when aggregate states are (z, μ), yet still unknown whether it will survive into next period. If the firm does not survive, then it chooses labor demand n, sells out its capital stock, and repays the debt to maximize the cash on hand x^d. The functional formulation is defined as

$$v_0(k, b, \varepsilon; z; \mu) = \pi_d \max_n [x^d(k, b, \varepsilon; z)] + (1 - \pi_d) v(k, b, \varepsilon; z; \mu), \tag{3}$$

where $x^d(k, b, \varepsilon, z) = z\varepsilon F(k, n) - w(z, \mu)n - b + q(1-\delta)k$. Conditional on survival, the continuing firm must choose current labor, next-period capital, and debt to maximize dividends. To clearly describe firms' investment behavior, I define the continuation problem as a binary choice between value function for upward and downward capital adjustment:

$$v(k, b, \varepsilon; z, \mu) = \max\{v^u(k, b, \varepsilon; z, \mu), v^d(k, b, \varepsilon; z, \mu)\}. \tag{4}$$

Denote that $d_g(z_f, \mu)$ is the stochastic discounting factor for the firm's next-period expected value if the realization of the exogenous aggregate state is z_g, given the current aggregate state is (z_f, μ). Taking the evolution of ε, z and $\mu' = \Gamma(z, \mu)$ as given, firms solve the optimization problems for both upward and downward capital adjustment by choosing labor demand, next-period capital, and next-period debt. The dynamic problem for upward-adjusting firms is to maximize the expected discounted dividend D such that (1) investment must be nonnegative, (2) borrowing limit is determined by its collateral, (3) dividend satisfies by nonnegative firm's budget constraint:

$$\begin{aligned} v^u(k, b, \varepsilon_i; z_f; \mu) = \max_{k', b', D} \; & D + \sum_{g=1}^{N_z} \pi^z_{fg} d_g(z_f; \mu) \sum_{j=1}^{N_s} \pi^\varepsilon_{ij} v_0(k', b', \varepsilon'_j; z'_g; \mu') \\ \text{s.t. } \; & 0 \le D \le x^u(k, b, \varepsilon; z_f) + q_b b' - Qk' \\ & x^u(k, b, \varepsilon_i; z_f) = z_f \varepsilon_i F(k, n) - w(z_f, \mu)n - b + Q(1-\delta)k \\ & k' \ge (1-\delta)k; \quad b' \le q\zeta k; \quad \mu' = \Gamma(z_f; \mu) \end{aligned} \quad , \tag{5}$$

where $x^u(\cdot)$ is the cash on hand for the upward-adjusting firm, q_b is the bond price, and ζ is a parameter for the efficiency of the economy's financial sector.

The downward-adjusting firms are different from the above problem only through (1) the investment must be nonpositive, and (2) firm's capital is evaluated by selling price q rather than purchasing price Q:

$$\begin{aligned} v^d(k, b, \varepsilon_i; z_f; \mu) = \max_{k', b', D} \; & D + \sum_{g=1}^{N_z} \pi^z_{fg} d_g(z_f; \mu) \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{ij} v_0(k', b', \varepsilon'_j; z'_g; \mu') \\ \text{s.t. } \; & 0 \le D \le x^d(k, b, \varepsilon_i; z_f) + q_b b' - qk' \\ & x^d(k, b, \varepsilon_i; z_f) = z_f \varepsilon_i F(k, n) - w(z_f, \mu)n - b + q(1-\delta)k \\ & k' \le (1-\delta)k; \quad b' \le q\zeta k; \quad \mu' = \Gamma(z_f; \mu) \end{aligned} \quad . \tag{6}$$

Since all firms are choosing labor demand regardless of continuation or not, given (k, ε), their decision rules on labor $N(k, \varepsilon; z, \mu)$ and output $Y(k, \varepsilon; z, \mu)$ does not depend on the current level of debt. On the contrary, the decisions on next-period capital and bond depend on all state variables, i.e., the decision rule on capital is $K(k, b, \varepsilon; z, \mu)$, and on the bond is $B(k, b, \varepsilon; z, \mu)$.

2.2 Household

My model economy is populated by a continuum of firms with unit measures. Let the flow utility function $u(c, 1-n^h) = \log(c) - \psi n^h$, the representative household maximize their lifetime utility by choosing consumption (c), labor supply (n^h), future firm shareholding (λ') and future bond holding (η'):

$$
\begin{aligned}
V^h(\lambda, \eta; z_f, \mu) = & \max_{c, n^h, \eta', \lambda'} \left\{ u(c, 1-n^h) + \beta \sum_{g=1}^{N_z} \pi^z_{fg} V^h(\lambda', \eta'; z'_g, \mu') \right\} \\
\text{s.t.} \quad & c + q(z_f; \mu)\eta' + \int_{\mathbf{S}} \rho_1(k', b', \varepsilon'_j; z'_g, \mu')\lambda(d[k' \times b' \times \varepsilon']) \ , \\
& \leq w(z_f; \mu)n^h + \eta + \int_{\mathbf{S}} \rho_0(k, b, \varepsilon; z_f \mu)\lambda(d[k \times b \times \varepsilon])
\end{aligned} \tag{7}
$$

where $\rho_0(\cdot)$ is the dividend-inclusive price of the current share, and $\rho_1(\cdot)$ is the ex-dividend price of the future share.

2.3 Recursive Equilibrium

A *recursive competitive equilibrium* is a set of functions,

$$
w, q, q_b, \{d_g\}_{g=1}^{N_z}, \rho_0, \rho_1, v_0, N, K, B, D, I, i_{new}, i_{used}, d, V^h, C^h, N^h, \eta^h, \Lambda^h \tag{8}
$$

such that

1. v_0 solves (3)-(6), and N is the corresponding policy functions for exiting firms, and (N, K, B, D) are the corresponding policy functions for continuing firms.
2. V^h solves (7), and (C^h, N^h, Λ^h) are the corresponding policy functions for households.
3. $\Lambda^h(k', b', \varepsilon'_j, \lambda, \eta; z_f, \mu) = \mu'(k', b', \varepsilon'_j; z_f, \mu)$ for all $(k', b', \varepsilon'_j) \in \mathbf{S}$.
4. Labor market clears:

$$
N^h(\lambda, \eta; z, \mu) = \int_{\mathbf{S}} [N(k, \varepsilon_i; z, \mu)]\mu(d[k \times b \times \varepsilon]), \tag{9}
$$

5. For upward-adjusting firms, i.e., firms such that $v^u(k,b,\varepsilon;z,\mu) \geq v^d(k,b,\varepsilon;z,\mu)$, the policy function $K(k,b,\varepsilon;z,\mu)$ solves (5), and the investment $I(k,b,\varepsilon;z,\mu) = K(k,b,\varepsilon;z,\mu) - (1-\delta)k$. Furthermore, the allocation of $i_{used}(k,b,\varepsilon;z,\mu)$ and $i_{new}(k,b,\varepsilon;z,\mu)$ is determined by the CES expenditure minimization problems.

$$\frac{i_{used}}{i_{new}} = \frac{1-\eta}{\eta}(q(z,\mu)+\gamma)^{-s}, \tag{10}$$

   and the corresponding aggregate price index is (2).

6. For downward-adjusting firms, i.e., $v^u(k,b,\varepsilon;z,\mu) < v^d(k,b,\varepsilon;z,\mu)$, the policy function $K(k,b,\varepsilon;z,\mu)$ solves (6), and $d(k,b,\varepsilon;z,\mu) = (1-\delta) - K(k,b,\varepsilon;z,\mu)$.

7. Good markets clear:

$$\begin{aligned} C(z,\mu) = \int_{\mathbf{S}} \Big\{ & z\varepsilon F(k, N(k,\varepsilon;z,\mu)) \\ & - (1-\pi_d)Q(z,\mu)I(k,b,\varepsilon;z,\mu) \\ & + (1-\pi_d)q(z,\mu)d(k,b,\varepsilon;z,\mu) \\ & + \pi_d[q(z,\mu)(1-\delta)k - k_0]\Big\}\mu(d[k\times b\times\varepsilon]) \end{aligned}, \tag{11}$$

   where k_0 is the initial capital stock. I assume k_0 for each entering firm is a fixed χ fraction of the long-run aggregate capital stock, i.e.,

$$k_0 = \chi \int k\tilde{\mu}(d[k\times b\times\varepsilon]), \tag{12}$$

   where $\tilde{\mu}$ represents the steady-state distribution.

8. The used investment price $q(z,\mu)$ clears the market of used capital:

$$\int_{\mathbf{S}} d(k,b,\varepsilon;z,\mu)\mu(d[k\times b\times\varepsilon]) = \int_{\mathbf{S}} i_{used}(k,b,\varepsilon;z,\mu)\mu(d[k\times b\times\varepsilon]). \tag{13}$$

9. Evolution of distribution $\Gamma(\mathbf{S},\mu)$ is defined by

$$\begin{aligned} \mu'(A,\varepsilon_i) = (1-\pi_d) & \int_{\{(k,b,\varepsilon)|K(k,b,\varepsilon;z,\mu),B(k,b,\varepsilon;z,\mu)\in A\}} \mu(d[k\times b\times\varepsilon]) \\ & + \pi_d\chi(k_0)H(\varepsilon_j) \end{aligned}, \tag{14}$$

   where $\chi(k_0) = 1$ if $(k_0, 0) \in A$, and 0 otherwise.

10. Bond market clear condition

$$\Phi^h(z,\mu) = \int_{\mathbf{S}} B(k,b,\varepsilon,z,\mu)\mu(d[k\times b\times\varepsilon]) \tag{15}$$

is satisfying Walras's law, where Φ^h is households' policy functions for bonds.

3 Analysis

To solve the recursive competitive equilibrium, I start to reformulate firms' problems by the optimality conditions implied by households' problems. Let C and N describe the market clearing consumption and labor supply, and C'_g and N'_g as the consumption and labor supply for the next period when aggregate TFP shock realization is $z' = z_g$. It is clear to show that (a) the real wage, $w(z,\mu)$, equals the marginal rate of substitution between leisure and consumption:

$$w(z,\mu) = \frac{D_2u(C,1-N)}{D_1u(C,1-N)}, \tag{16}$$

(b) the inverse of bond price, $q_b^{-1}(z_f,\mu)$, equals to the expected gross real interest rate:

$$q_b(z_f,\mu) = \beta\sum_{g=1}^{N_z}\pi^z_{fg}\frac{D_1u(C'_g,1-N'_g)}{D_1u(C,1-N)}, \tag{17}$$

(c) the state-contingent discount factor, $d_g(z_f,\mu)$, equals to the intertemporal rate of substitution across states:

$$d_g(z_f,\mu) = \beta\frac{D_1u(C'_g,1-N'_g)}{D_1u(C,1-N)}. \tag{18}$$

By applying the above three conditions, I merged the decision rule in households' problems with firms' problems.

Without the loss of generality, I assign $p(z,\mu)$ to be the household's marginal utility of consumption. The p function represents the output price in terms of households' marginal utility. It allows firms to discount their current dividend and payment by households' subjective discounting factor. I can rewrite (16)-(18) as

$$p(z_f,\mu) = D_1u(C,1-N) = \frac{1}{C}, \tag{19}$$

$$w(z_f,\mu) = \frac{D_2u(C,1-N)}{p(z_f,\mu)} = \frac{\psi}{p(z_f,\mu)}, \tag{20}$$

$$q_b(z_f,\mu) = \beta\sum_{g=1}^{N_z}\pi^z_{fg}\frac{p(z_g,\mu')}{p(z_f,\mu)}, \tag{21}$$

Following Khan and Thomas (2013) and the definition of $p(z_f, \mu)$, I can rewrite equations (3)-(6) as

$$V_0(k, b, \varepsilon; z, \mu) = \pi_d \max_n [p(z, \mu) x^d(k, b, \varepsilon; z)] + (1 - \pi_d) V(k, b, \varepsilon; z, \mu), \tag{22}$$

where

$$V(k, b, \varepsilon; z, \mu) = \max\{V^u(k, b, \varepsilon; z, \mu), V^d(k, b, \varepsilon; z, \mu)\}. \tag{23}$$

The dynamic problem for upward-adjusting firms is

$$\begin{aligned} V^u(k, b, \varepsilon; z, \mu) = \max_{k', b', D} & \; p(z, \mu) D + \beta \sum_{g=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi^z_{fg} \pi^\varepsilon_{ij} V_0(k', b, \varepsilon'_j; z'_g, \mu') \\ \text{s.t. } & 0 \leq D \leq x^u(k, b, \varepsilon; z) + q_b b' - Qk' \\ & x^u(k, b, \varepsilon; z) = z\varepsilon F(k, n) - w(z, \mu) n - b + Q(1 - \delta) k \\ & k' \geq (1 - \delta) k; \quad b' \leq q\zeta k; \quad \mu' = \Gamma(z; \mu) \end{aligned}, \tag{24}$$

and the dynamic problem for downward-adjusting firms is

$$\begin{aligned} V^d(k, b, \varepsilon; z, \mu) = \max_{k', b', D} & \; p(z, \mu) D + \beta \sum_{g=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi^z_{fg} \pi^\varepsilon_{ij} V_0(k', b', \varepsilon'_j; z'_g, \mu') \\ \text{s.t. } & 0 \leq D \leq x^d(k, b, \varepsilon; z) + q_b b' - qk' \\ & x^d(k, b, \varepsilon; z) = z\varepsilon F(k, n) - w(z, \mu) n - b + q(1 - \delta) k \\ & k' \leq (1 - \delta) k; \quad b' \leq q\zeta k; \quad \mu' = \Gamma(z; \mu) \end{aligned}. \tag{25}$$

To determine the decision of (k', b', D) for continuing firms, I impose the *minimum saving policy* to simplify the analysis and to synthesize the choice in both intertemporal assets. Following the definition in Khan and Thomas (2013), the definition of *constrained* firm is that firms are facing binding collateral constraints with nonzero possibility for any possible current or future state. Otherwise, the firm is *unconstrained*.

The constrained firms in this definition may not face a binding collateral constraint in the current period but might face a binding collateral constraint in any possible future state. For constrained firms, since their marginal rate of intertemporal substitution exceeds the current valuation of their dividends, $p(\cdot)$, they can only set $D = 0$, and the choice of k' is implied by $b' = q\zeta k$ and the binding budget constraint. Thus, the problem faced by a constrained firm becomes a univariate problem. For unconstrained firms, their dividend and the financial saving decision will ensure that the existence of collateral constraints will never affect their capital accumulation. Therefore, the return on financial saving and capital accumulation should equal the household's evaluation, $p(\cdot)$, in equilibrium. With this observation in mind, I can solve the efficiency unit of capital given the idiosyncratic productivity of each firm and aggregate states. The above process solves the capital decisions. For decisions on financial saving and dividends, I follow Khan and Thomas (2013) and assume the *minimum saving policy*, i.e., unconstrained firms will prioritize dividend spending rather than financial saving, and only put extra profit into saving only to ensure its unconstrained status.

Let W be the unconstrained firm's value and W_0 be the expected value of the firm before the exogenous exit is realized. These two functions are counterparts of equation (22) and (23), and similarly formated:

$$W_0(k,b,\varepsilon;z,\mu) = \pi_d \max_n [p(z,\mu)x^d(k,b,\varepsilon;z)] + (1-\pi_d)W(k,b,\varepsilon;z,\mu), \tag{26}$$

and

$$W(k,b,\varepsilon;z,\mu) = \max\left\{W^u(k,b,\varepsilon;z,\mu), W^d(k,b,\varepsilon;z,\mu)\right\}. \tag{27}$$

Since (i) the bond choice b' and capital choices k' of unconstrained firms are orthogonal to each other, and (ii) their marginal payoff to the firm can be represented by the household's valuation $p(\cdot)$, I can express $W(k,b,\varepsilon;z,\mu) = W(k,0,\varepsilon;z,\mu) - pb$ and $W_0(k,b,\varepsilon;z_f,\mu) = W_0(k,0,\varepsilon;z,\mu) - pb$. Given these transformations, I have

$$\begin{aligned} W^u(k,b,\varepsilon_i;z_f,\mu) &= p(z_f,\mu)x^u(k,b,\varepsilon_i;z_f) \\ &\quad + \max_{k' \geq (1-\delta)k} \left[-pQk' + \beta \sum_{g=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi^z_{fg}\pi^\varepsilon_{ij} W_0(k',0,\varepsilon_j;z_g,\mu)\right], \end{aligned} \tag{28}$$

where $x^u(k,b,\varepsilon_i;z_f) = z_f\varepsilon_i F(k,n) - w(z_f,\mu)n - b + Q(1-\delta)k$, and

$$\begin{aligned} W^d(k,b,\varepsilon_i;z_f,\mu) &= p(z_f,\mu)x^d(k,b,\varepsilon_i;z_f) \\ &\quad + \max_{k' \leq (1-\delta)k} \left[-pqk' + \beta \sum_{g=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi^z_{fg}\pi^\varepsilon_{ij} W_0(k',0,\varepsilon_j;z_g,\mu)\right], \end{aligned} \tag{29}$$

where $x^d(k,b,\varepsilon_i;z_f) = z_f\varepsilon_i F(k,n) - w(z_f,\mu)n - b + q(1-\delta)k$.

The upward- and downward-adjustment capital target, $k^*_u(\varepsilon_i;z_f,\mu)$ and $k^*_d(\varepsilon_i;z_f,\mu)$, are the solutions of (28) and (29), respectively, and hence the capital decision rule of unconstrained firms follows (S, s) form:

$$K^w(k,\varepsilon;z_f,\mu)=\begin{cases}k_u^*(\varepsilon;z_f,\mu) & \text{if } k_u^*(\varepsilon;z_f,\mu)>(1-\delta)k\\(1-\delta)k & \text{if } (1-\delta)k\in[k_d^*(\varepsilon;z_f,\mu),k_u^*(\varepsilon;z_f,\mu)]\\k_d^*(\varepsilon;z_f,\mu) & \text{if } k_d^*(\varepsilon;z_f,\mu)<(1-\delta)k\end{cases}. \tag{30}$$

The minimum saving policy, $B^w(k,\varepsilon;z_f,\mu)$, is derived recursively by searching over all possible largest debt levels, $\tilde{B}(K^w(\cdot),\varepsilon_j;z_g,\mu')$, which ensures firms to remain in an unconstrained status entering the next period. If the firm chooses its future debt level $B^w(\cdot)$ as the minimum of all possible $\tilde{B}(\cdot)$, then firms are paying the largest amount of dividends without any possibility of losing its unconstrained status in the future:

$$B^w(k,\varepsilon;z_f,\mu)=\min_{\{\varepsilon_j|\pi^\varepsilon_{ij}>0 \text{ and } z_g|\pi^z_{fg}>0\}}\tilde{B}(K^w(k,\varepsilon_i;z_f,\mu),\varepsilon_j;z_g,\mu'), \tag{31}$$

where $\tilde{B}(k,\varepsilon_i;z_f,\mu)$ is defined as the highest current debt that a firm can take without violating the collateral constraints:

$$\begin{aligned}\tilde{B}(k,\varepsilon;z,\mu)=&\ z\varepsilon F(k,N(k,\varepsilon))-wN(k,\varepsilon)\\&+q_b\min\{B^w(k,\varepsilon;z,\mu),q\zeta k\}\\&+\mathcal{J}(K^w(k,\varepsilon)-(1-\delta)k)[K^w(k,\varepsilon;z,\mu)-(1-\delta)k]\end{aligned}. \tag{32}$$

where $\mathcal{J}(x)=Q$ if $x\geq 0$, and $\mathcal{J}(x)=q$ if $x<0$. Given the decision on bond and capital, I can retrieve unconstrained firms' dividend payments as

$$D^w(k,b,\varepsilon;z_f,\mu)\begin{cases}x^u(k,b,\varepsilon_i;z_f)-QK^w(k,\varepsilon) & \\ \quad +q_b\min\{B^w(k,\varepsilon;z_f,\mu),q\zeta k\} & \text{if } K^w(k,\varepsilon)\geq(1-\delta)k\\ x^d(k,b,\varepsilon_i;z_f)-qK^w(k,\varepsilon) & \\ \quad +q_b\min\{B^w(k,\varepsilon;z_f,\mu),q\zeta k\} & \text{if } K^w(k,\varepsilon)<(1-\delta)k\end{cases}. \tag{33}$$

Constrained firms are also facing exogenous exits.

$$V_0(k,b,\varepsilon;z,\mu)=\pi_d\max_n[p(z,\mu)x^d(k,b,\varepsilon;z)]+(1-\pi_d)V(k,b,\varepsilon;z,\mu). \tag{34}$$

Conditional on their survival, they will adopt unconstrained firms' decision rule and gain unconstrained status if they can do so, i.e., they can adopt the minimum saving policy and pays $D^w(k,b,\varepsilon;z,\mu)$:

$$V(k,b,\varepsilon;z,\mu)=\begin{cases}W(k,b,\varepsilon;z,\mu) & \text{iff } D^w(k,b,\varepsilon;z,\mu)\geq 0\\ V^c(k,b,\varepsilon;z,\mu) & \text{otherwise}\end{cases}. \tag{35}$$

If they cannot achieve $W(\cdot)$, then they borrow up to collateral constraint and approach the efficiency unit of capital as closely as possible by choosing to conduct upward- or downward-adjustment of capital:

$$V^c(k,b,\varepsilon;z,\mu)=\max\{V^u(k,b,\varepsilon;z,\mu),V^d(k,b,\varepsilon;z,\mu)\}, \tag{36}$$

Since by definition constrained firms prioritize enhancing their status to unconstrained by accumulating financial wealth and achieving efficiency unit of capital, they pay no dividend. Thus, their objective is the expected future value of the firms, and their debt level is implied by their capital decision with binding budget constraint, that is

$$\begin{aligned}V^u(k,b,\varepsilon_i;z_f,\mu)&=\max_{k'\in\Omega^u(k,b,\varepsilon)}\beta\sum_{g=1}^{N_z}\sum_{j=1}^{N_\varepsilon}\pi^z_{fg}\pi^\varepsilon_{ij}V_0(k',b'_u(k'),\varepsilon_j;z_g,\mu')\\ \text{subject to } b'_u(k')&=\frac{Qk'-x^u(k,b,\varepsilon;z_f)}{q_b}\end{aligned}, \tag{37}$$

and

$$\begin{aligned}V^d(k,b,\varepsilon_i;z_f,\mu)&=\max_{k'\in\Omega^d(k,b,\varepsilon)}\beta\sum_{g=1}^{N_z}\sum_{j=1}^{N_\varepsilon}\pi^z_{fg}\pi^\varepsilon_{ij}V_0(k',b'_d(k'),\varepsilon_j;z_g,\mu')\\ \text{subject to } b'_d(k')&=\frac{qk'-x^d(k,b,\varepsilon;z_f)}{q_b}\end{aligned}. \tag{38}$$

If the debt decision, $b'_u(\cdot)$ and $b'_d(\cdot)$, ever reaches the collateral constraint, then they define the endogenous maximum affordable capital stocks for each option:

$$\begin{aligned}b'_u(k')=q\zeta k=\frac{Qk'-x^u(k,b,\varepsilon;z_f)}{q_b}&\Rightarrow\bar{k}_u=\frac{q_bq\zeta k+x^u}{Q}\\ b'_d(k')=q\zeta k=\frac{qk'-x^d(k,b,\varepsilon;z_f)}{q_b}&\Rightarrow\bar{k}_d=\frac{q_bq\zeta k+x^d}{q}\end{aligned}. \tag{39}$$

Thus, the endogenous limit, $\bar{k}_u$ and $\bar{k}_d$, construct the choice sets, $\Omega^u(k,b,\varepsilon)$ and $\Omega^d(k,b,\varepsilon)$, of upward- and downward-adjusting problem (37) and (38):

$$
\begin{aligned}
\Omega^u(k,b,\varepsilon) &= [(1-\delta)k, \bar{k}_u(k,b,\varepsilon)] \\
\Omega^d(k,b,\varepsilon) &= [0, \max\{0, \min\{(1-\delta), \bar{k}_d(k,b,\varepsilon)\}\}]
\end{aligned}. \tag{40}
$$

Let the solution for (37) and (38) be $\hat{k}_u(k,b,\varepsilon)$ and $\hat{k}_d(k,b,\varepsilon)$, the policy function for capital is

$$
K^c(k,b,\varepsilon;z_f,\mu) = \begin{cases} \hat{k}_u(k,b,\varepsilon) & \text{if } V(\cdot) = V^u(\cdot) \\ \hat{k}_d(k,b,\varepsilon) & \text{if } V(\cdot) = V^d(\cdot) \end{cases}, \tag{41}
$$

and the corresponding policy function for the bond is

$$
B^c(k,b,\varepsilon;z_f,\mu) = \begin{cases} \frac{Q\hat{k}_u(k,b,\varepsilon)-x^u}{q_b} & \text{if } V(\cdot) = V^u(\cdot) \\ \frac{q\hat{k}_d(k,b,\varepsilon)-x^d}{q_b} & \text{if } V(\cdot) = V^d(\cdot) \end{cases}. \tag{42}
$$

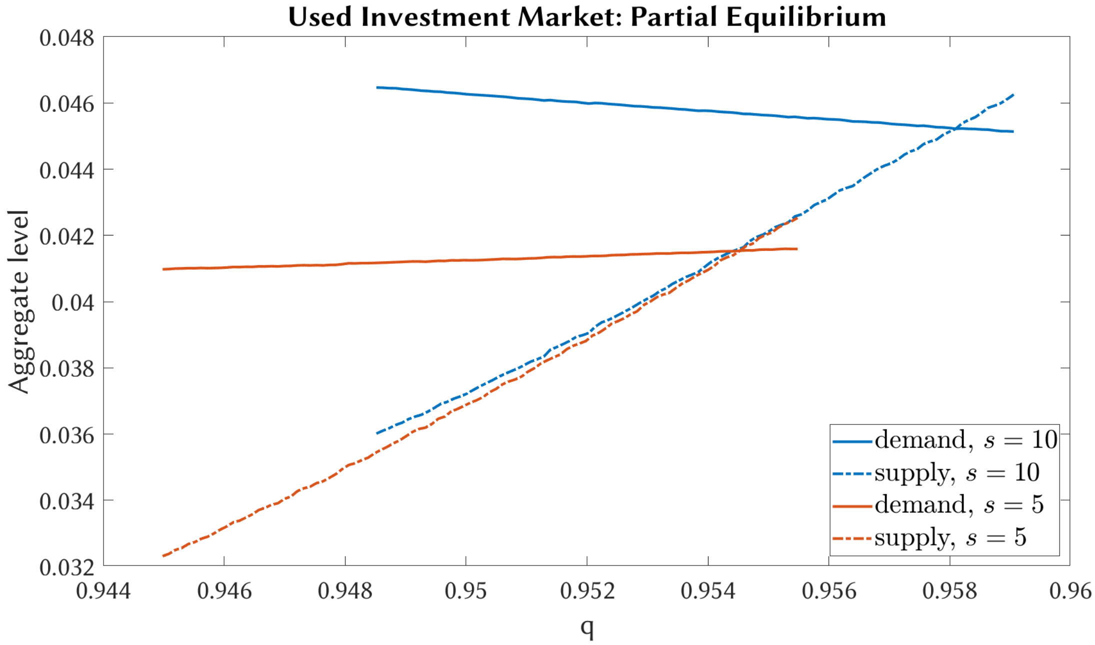
4 Steady State Calibration

Table 1 lists the parameter values, and table 2 summarizes the matching of aggregate moments between the model and data. Since capital investment is the core mechanics of this model, I set the length of a period to one year to match the establishment-level investment data. I assume that the functional form of the representative household's utility is following the specification in Rogerson (1988), i.e., $u(c, l) = \log c + \psi l$. The production function is Cobb-Douglas: $z\varepsilon F(k, n) = z\varepsilon k^{\alpha} n^{\nu}$. The model exhibits exogenous entry and exit to keep all firms from outgrowing the collateral constraint. Entrants are endowed with initial capital k_0 as a fraction of steady-state aggregate capital, as specified in (12), and initial bond level $b_0 = 0$. The household's discount rate β is set to imply 4 percent of the annual interest rate. The disutility from working, ψ, is determined to reproduce hours of work equal to one-third. The rate of capital depreciation, δ, corresponds to an investment-capital ratio of approximately 10 percent. The labor share ν is 60 percent, as shown in US postwar data.

In the steady state, the aggregate productivity is a constant, $z = 1$. I assume that the idiosyncratic productivity shock follows AR(1) process in logs with autocorrelations ρ_ε and standard deviations σ_ε, $\log \varepsilon' = \rho_\varepsilon \log \varepsilon + \eta'_\varepsilon$, with $\eta'_\varepsilon \sim N(0, \sigma^2_{\eta_\varepsilon})$. The parameters for idiosyncratic productivity, ρ_ε and $\sigma_{\eta_\varepsilon}$ are calibrated jointly with the reallocation cost γ to match three moments in the establishment-level investment distribution of Cooper and Haltiwanger (2006), namely standard deviation of investment rates (i/k), $\sigma(i/k) = 0.337$; the average serial correlation of investment rates, $\rho(i/k) = 0.058$; and frequency of lumpy investment, 0.186. The model-generated moments are $\sigma(i/k) = 0.387$, $\rho(i/k) = 0.022$, and frequency of lumpy investment 0.171, and the resulting parameters are $\sigma_{\eta_\varepsilon} = 0.659$, $\rho_\varepsilon = 0.118$ and $\gamma = 0.026$. With these parameters in hand, I use Rouwenhorst (1995) method to discretize the firm's log-normal idiosyncratic productivity process with 7 values ($N_\varepsilon = 7$) to obtain $\{\varepsilon_i\}_{i=1}^{N_\varepsilon}$ and $(\pi^{\varepsilon}_{ij})_{i,j=1}^{N_\varepsilon}$. The equilibrium degree of capital irreversibility, i.e., q/Q, is 0.9612, close to the degree of irreversibility in Khan and Thomas (2013), which is 0.954. This model also matches closely with the average mean investment rate from Cooper and Haltiwanger (2006). That moment in data is 0.122, and my model counterpart is 0.120.

The investment technology is determined by two parameters: the average ratio of two investments, η, and the elasticity of substitution s. Following Lanteri (2018), I calibrate the parameter η to match the capital reallocation to investment ratio in Eisfeldt and Rampini (2006), 0.2389. The calibrated value for η is 0.85, and the corresponding reallocation-to-investment ratio is 0.169. The elasticity of substitution between two types of investment goods s is set to 10 in the benchmark case, which is within the range estimation provided by Edgerton (2011), $s \in [1, 10]$. There is an numerical limitation on the choice of s. As

Figure 1: Partial Equilibrium Analysis: Used Investment Market with different CES parameter s



shown in the figure 1, when the value of CES parameter $s = 5$, the demand curve is upward-sloping. If the CES parameter $s = 8$, then the demand curve is roughly flat. Therefore, to get a stable convergence in the perfect foresight exercise, I choose $s = 10$ to guarantee a downward-sloping demand curve.

Following the Khan and Thomas (2013) algorithm, I recursively solve this model. Given a combination of wage $w(z_f, \mu)$ and used investment price $q(z_f, \mu)$, I solve the unconstrained firm's problem by searching all (k, ε) states and find the capital decision rule for an unconstrained firm $K^w(k, \varepsilon; z_f, \mu)$ in (30). Later, find the minimum saving policy $B^w(k, \varepsilon; z_f, \mu)$ in (31) and (32) for firm identified as (k, ε) to remain unconstrained status as well as to prioritize paying the dividend. If the current debt level b is lower than the $\tilde{B}(k, \varepsilon_i; z_f, \mu)$, then we call the firm at (k, b, ε) is in unconstrained status. On the contrary, if $b > \tilde{B}(k, \varepsilon_i; z_f, \mu)$, then the firm at (k, b, ε) is constrained. As a result, there are over 80 percent of the state space is unconstrained status. For the rest of the constrained firms, I solve the constrained firm's problem in (34)-(40) to find the policy function for capital $K^c(k, b, \varepsilon; z_f, \mu)$ in (41) and its implied $B^c(k, b, \varepsilon; z_f, \mu)$ in (42). The next step is to calculate the stationary distribution given the policy functions for both types of firms. I linearly interpolate the policy functions on (k, b, ε) to compute firms' decision rules on stationary distribution. Notice that to accelerate the speed of convergence, I move those unconstrained firms to their own (k, ε) distribution,

Table 1: Calibration

Parameter	Description	Value
Preferences and technology		
β	Subjective discount factor	0.96
ψ	Disutility from working	2.15
α	Capital share	0.270
ν	Labor share	0.600
δ	Depreciation rate	0.065
Shocks		
ρ_ε	Persistence idiosyncratic productivity shock	0.659
$\sigma_{\eta_\varepsilon}$	Volatility idiosyncratic productivity shock	0.118
Firm characteristic		
ζ	efficiency of the financial sector	1.2
π_d	exogenous exit probability	0.1
χ	fraction of entrants' capital endowment to aggregate capital	0.1
Investment technology		
η	new investment ratio	0.85
s	elasticity of substitution between new and used investment	10.0
γ	installation cost of used investment	0.026
Equilibrium Prices		
q/Q	degree of capital irreversibility	0.9572
q	used investment selling price	0.9541
Q	effective capital purchasing price	0.9967

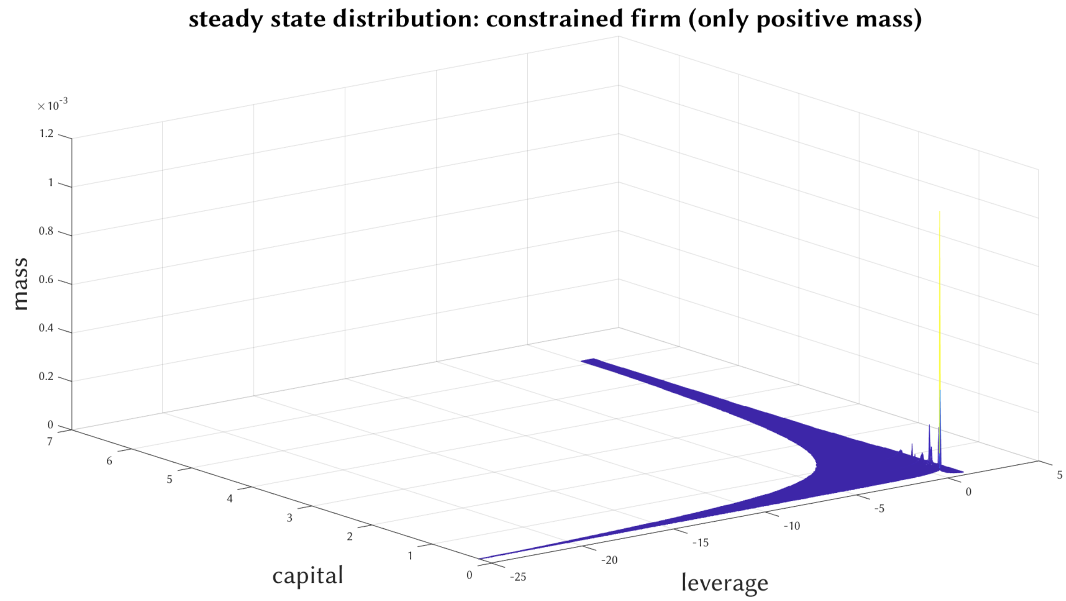
and only the constrained firms' distribution needs to iterate on (k, b, ε) to track their current bond state. Lastly, I use the two-dimensional bisection method to approach equilibrium $w(z_f, \mu)$ and $q(z_f, \mu)$. This bisection method is my simplification based on Harvey and Stenger (1976). Harvey-Stenger bisection method contains two parts: (i) generate a polygon that is large enough to contain roots, and (ii) bisect on polygon and find triangles containing roots, and keep bisecting triangles until convergence. The simplification I made is twofold. First, I replace the polygon generation with a test on whether the initial triangle contains roots. Second, instead of directly evaluating the functional value of the centroid, I linearly interpolate the mid-point on the longest side and the vertex across from the longest side. This reuses the three functional values, and only evaluates the mid-point on the longest side in each iteration.

Table 2: Aggregate Moments

	model	data
First moments		
Capital/Output, K/Y	2.275	2.39
Debt/Capital, B/K	0.367	0.37
Labor share, wN/Y	0.599	0.6
Investment/Capital, I/K	0.069	0.069
Reallocation/Investment	0.1699	0.2389
Second moments		
standard deviation of investment rate, $\sigma(i/k)$	0.387	0.337
serial correlation of investment rate, $\rho(i/k)$	0.022	0.058
frequency of lumpy investment ($i/k > 20\%$)	0.171	0.186
Untargeted moments		
average mean of investment rate, $\mu(i/k)$	0.120	0.122
frequency of inaction region ($abs(i/k) < 1\%$)	0.472	0.081
frequency of negative investment	0.146	0.104
frequency of lumpy disinvestment ($i/k < -20\%$)	0.095	0.018

Note: In Eisfeldt and Rampini (2006), Reallocation is the sum of acquisitions and sales of PP&E, i.e., the aggregate expense on used capital. Investment is the sum of acquisitions, sales of PP&E, and new capital expense.

Figure 2: Constrained firm steady-state distribution: median productivity

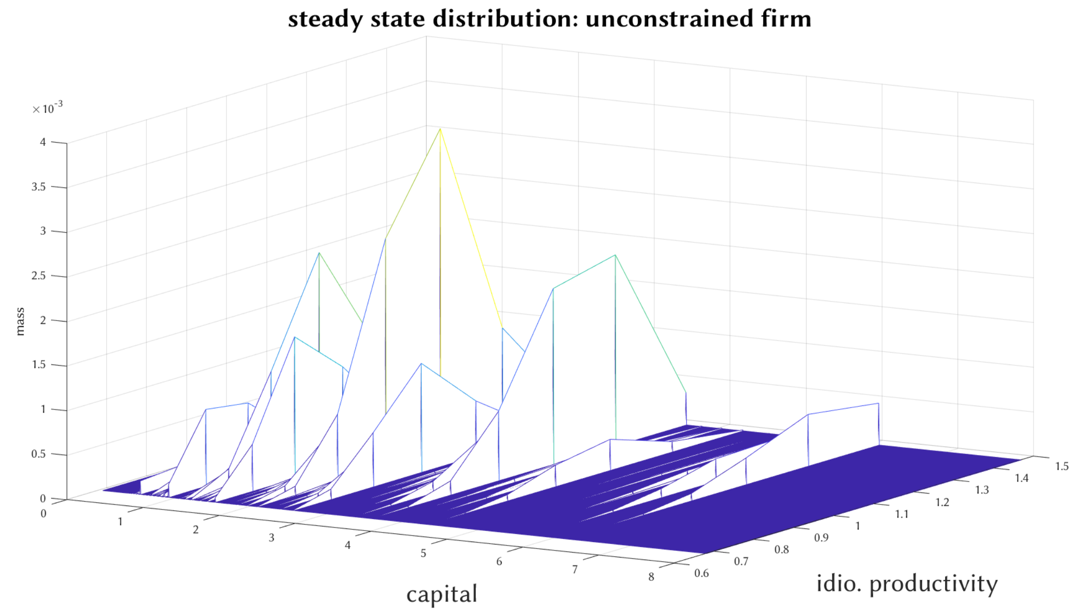


5 Steady State Results

Figures 2 and 3 contain the distribution of constrained and unconstrained firms with median productivity. Within Figure 3, there are also no-constraint firms contained in the unconstrained distribution. Distributions over capital and leverage at other productivity levels are similar to Figures 2 and 3. The idiosyncratic productivity of the 10 percent of entrants are following the ergodic distribution $(\pi_i^{\varepsilon})_{i=1}^{N_\varepsilon}$. They are endowed with zero debt and 10 percent of the steady-state aggregate capital, 0.130. The spike in figure 2 depicts these entrants. Once entrants start production, they take debt to accumulate their capital. In absence of collateral constraint, they would immediately take a large and temporary debt to reach the efficiency unit of capital determined by their idiosyncratic productivity draw. Nevertheless, firms with little capital have limited ability to borrow, forcing them to gradually accumulate their capital as they age. This smooth growth of capital is represented by the series of lower spikes following the spike of new entrants alongside the k-dimension. Those surviving firms who live long enough will reach the targeted capital and reduce their debt or accumulate financial savings, corresponding to the extended long tails toward negative leverage at each little spike in figure 2. Eventually, 5.9 percent of firms retained enough financial savings and entered the unconstrained status, while 94.1 percent of firms remained at constrained status.

An analysis of the steady state is also necessary to understand firms' decisions in my model. Figure 4 shows the life cycle aspect of my model through simulation. I simulate

Figure 3: Unconstrained firm steady-state distribution: median productivity



50, 000 firms over 100 periods without exogenous entry and exit to get a large panel for established firms as seen in Compustat data. After the panel data in hand, I calculate the average capital and leverage of over 50, 000 in firms at each period. In the initial 5 periods, firms are accumulating capital by raising debt. Starting at period 7, they begin to reduce their debt yet still accumulate capital. By period 16, the firm has become a net saver, and eventually reach its desired leverage level at period 35. From the above discussion regarding figure 4, financial imperfection has hindered the optimal investment responses by limiting the available funding. Any firm can take 120% of leverage, yet as we can see in the simulation, financial imperfection still generates capital misallocation. The cause of the misallocation comes from not the over-investment of unconstrained firms, but the under-investment of the constrained firm. In the stationary distribution, the average capital among unconstrained firms is 2.13, while the average capital among constrained firms is 1.24, with 34 percent of firms facing binding collateral constraint.

The steady-state results are very similar to the results in Khan and Thomas (2013). The main difference is that investing one unit of effective capital requires only $Q = 0.9967$ of consumption goods. Compared to my replication of Khan and Thomas (2013) model, opening used capital market results in a 0.8 percent increase in steady state output yet a 0.074 percent decrease in measured TFP, which is measured by Solow residual. With lower investment costs, it is easier for firms to invest, yet the aggregate capital increased by 1.977

Figure 4: Cohort in steady state: life cycle simulation

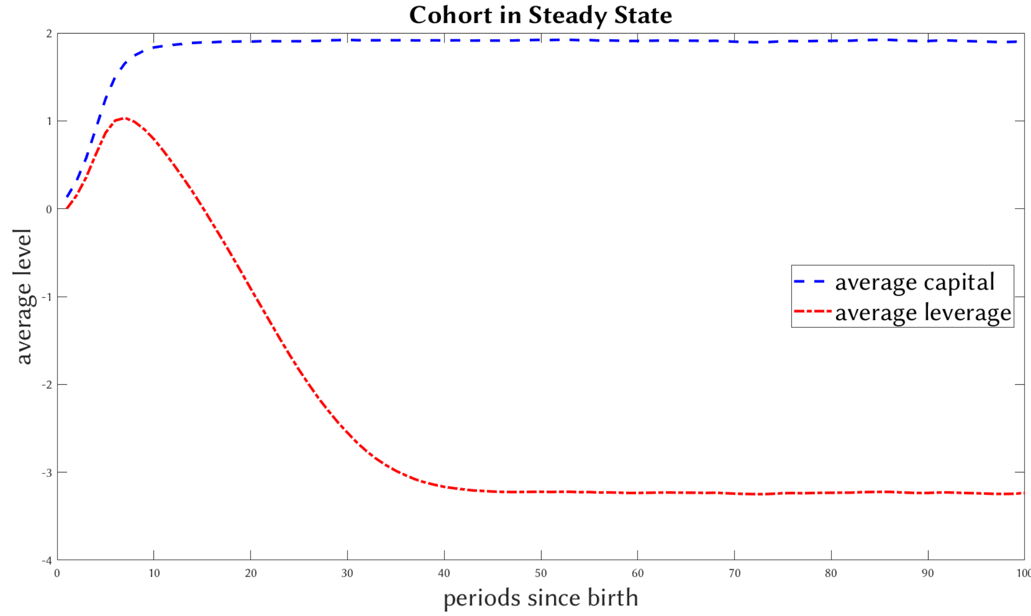


Figure 5: Investment Rate Distribution

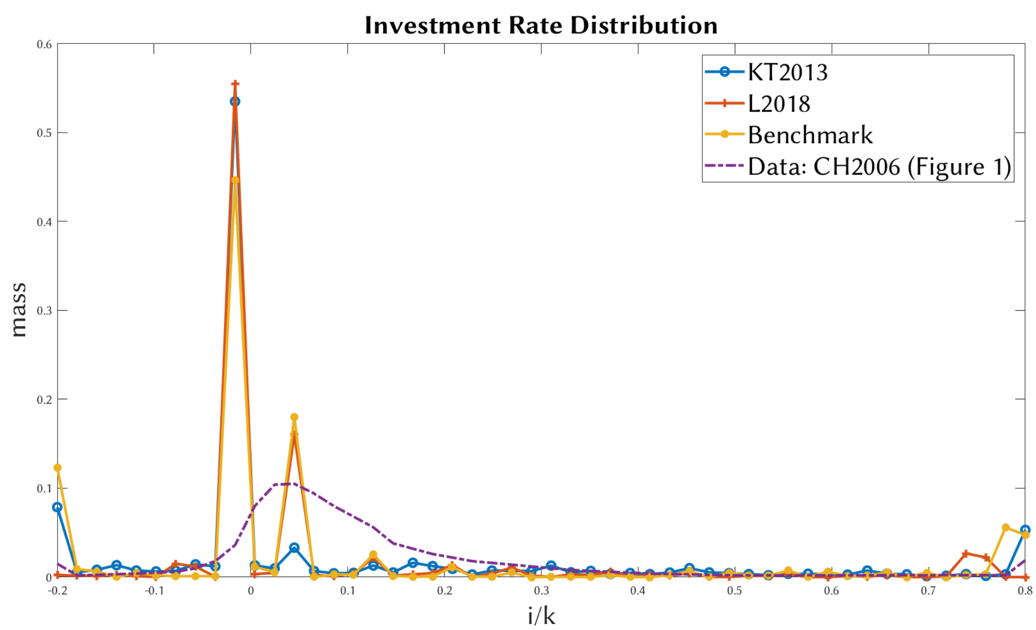
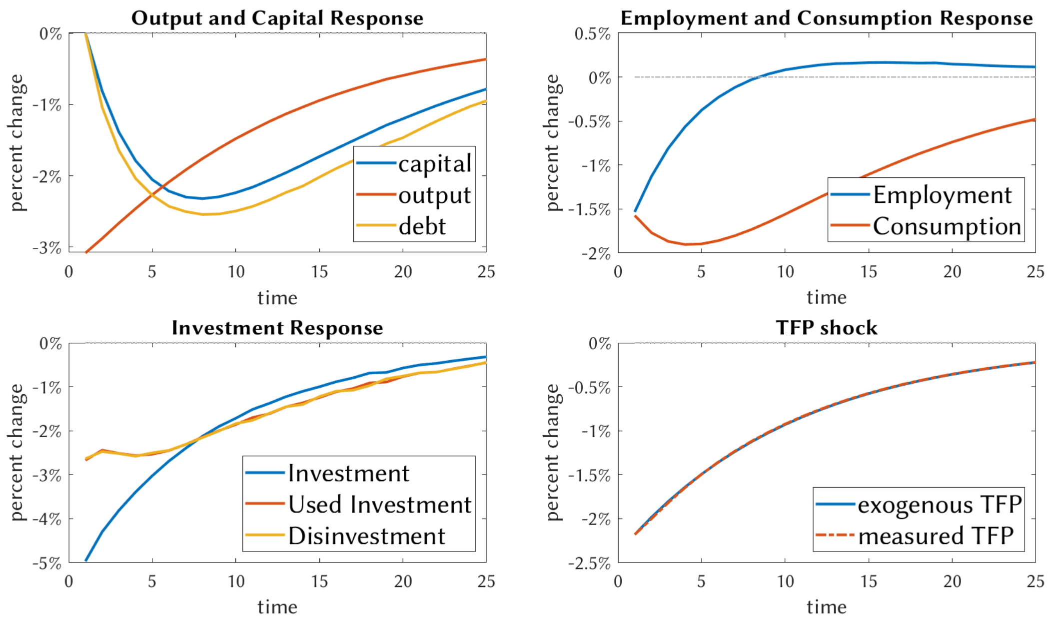


Figure 6: Impulse Response to 2.18% decrease in TFP with persistence $\rho_{\eta_z} = 0.909$



percent, resulting in a decrease in measured TFP. Note that even though it might seem to be a small change compared to Khan and Thomas (2013) in steady state, the procyclical used capital price q and effective capital purchasing price Q will amplify the impact of both real and financial frictions in this model economy.

Figure 6 featured the impulse response to an unexpected 2.18% drop in TFP z with persistence $\rho_{\eta_z} = 0.909$. This figure has shown that my model is able to match two empirical facts: (1) capital reallocation is procyclical, and (2) debt financing and capital reallocation is positively correlated. The aggregate response is similar to Hansen (1985) and Khan and Thomas (2013). As illustrated in the Lanteri (2018), having the investment technology specified in (1) is able to generate procyclical capital reallocation, which is represented by the Disinvestment and Used Investment sequence. With the addition of debt, my model also matches the strong positive correlation between debt financing and capital reallocation. Figure 7 shows that the used capital price is procyclical.

Figure 8 and 9 shows that the used capital market serves as a automatic stabilizer in response to a 37.5% drop in credit parameter ζ. This drop in ζ in steady state causes 26% of drop in debt, yet in a perfect foresight transition, the trough is only 18% of decrease in debt. Moreover, the used capital price q increases more than 1% when the firms are expected the credit parameter ζ to recover at the rate of $1 - \rho_z$, indicating that the used capital become more attractive when firms are more confined in terms of bond borrowing.

Figure 7: Impulse Response to 2.18% decrease in TFP with persistence $\rho_{\eta_z} = 0.909$, Price Reaction

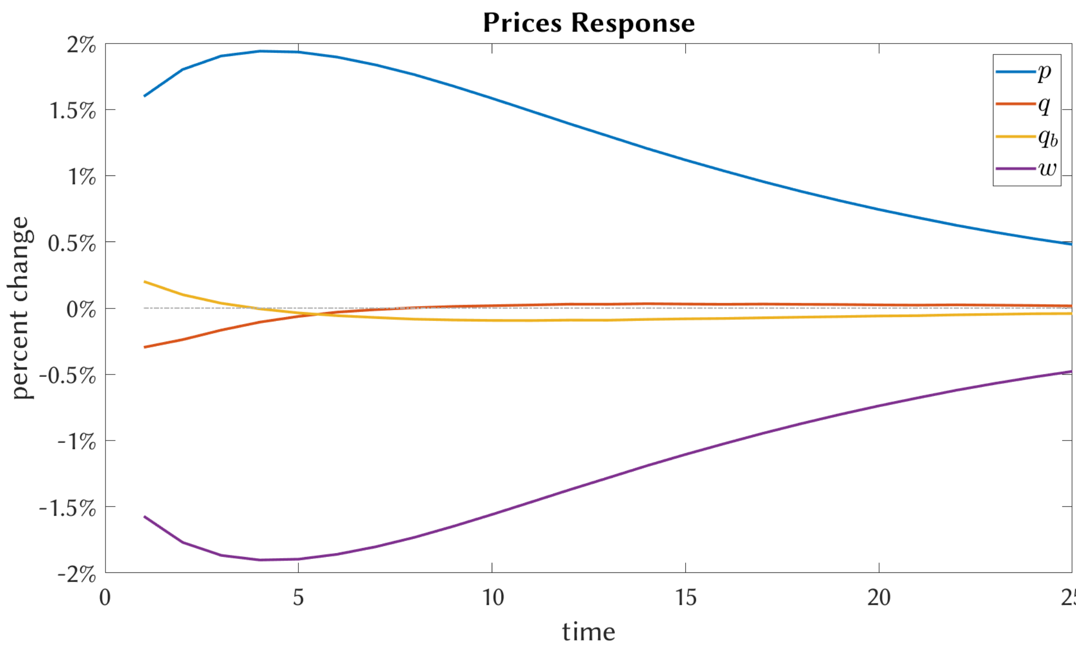


Figure 8: Impulse Response to 37.5% decrease in credit with persistence $\rho_{\eta_z} = 0.909$

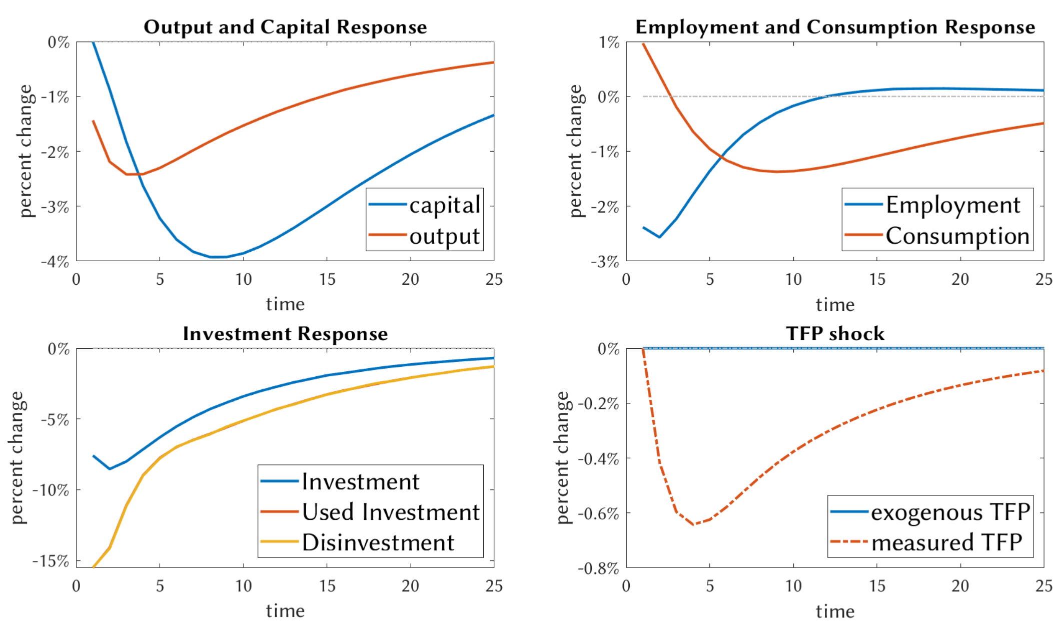


Figure 9: Impulse Response to 37.5% decrease in credit with persistence $\rho_{\eta_z} = 0.909$: Price and Debt Reaction

6 Concluding Remarks

I have developed a dynamic stochastic general equilibrium model with persistent idiosyncratic shock, endogenous capital irreversibility, and collateralized borrowing constraints based on market value. I have calibrated the model to match aggregate and firm-level data. My model economy generates a sophisticated distribution of firms over productivity, capital, and bond that determines the aggregate output and investment.

Firms take the used investment price as given, and respond endogenously to the capital selling and purchasing price. Over time, they build precautionary saving to ensure that collateral constraint does not affect their optimal investment decision, yet only a fraction of firms can be free from financial imperfection. The majority of young and small firms shaped the used investment price with two opposing forces. A cheaper effective capital price induces firms to invest more, while the lower market value of their capital limits their available funding. In equilibrium, the aggregate investment increased, and the equilibrium irreversibility of capital is lower than the models in the literature.

The next step of this research is to incorporate aggregate fluctuation into the model and evaluate how the used investment price reacts. After the Great Recession, there is an unusually slow recovery of investment and employment over 18 months since $Q2$ of 2009. The procyclical borrowing limit in my model may be able to reproduce such a slow recovery in investment by the interaction between credit shock and used investment price. As a credit crunch happens, the funding for investing firms drops, causing the used investment price to plummet. The inaction firm caused by high capital irreversibility may not immediately respond to the recovery of credit condition, leading to insufficient demand to push the used investment price back. As the price stays low, more firms stay at binding collateral constraints, causing a slow recovery in investment.

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